Launching an object from surface of Earth – how high does it go?

Note Title 11/18/2010

Keep in mind $r = h + R_E$ and $\frac{1}{2}mv_0^2 - \frac{GmM_E}{r_0} = \frac{1}{2}mv_f^2 - \frac{GmM_E}{r_f}$

and $GM_E = 4\times10^{14}$

if start at surface and launch: $r_0 = R_E$ $\quad r_f = R_E + h$ $\quad v_f = 0$

take energy equation – multiply through by 2 and divide by m:

$$v_0^2 - \frac{2GM_E}{R_E} = \frac{2GM_E}{r_f}$$

hey $\frac{2GM_E}{R_E} = v_{esc}^2 = (11200\ \text{m/sec})^2$

$$v_0^2 - v_{esc}^2 = \frac{2(4\times10^{14})}{r_f}$$

or

$$r_f = \frac{2(4\times10^{14})}{v_0^2 - v_{esc}^2}$$

$v_{esc} = 11200\ \text{m/sec}$

Please consider using $GM_E = 4\times10^{14}$ ... much easier than dealing with G and M_E!

Sometimes using variables in the calculation (instead of the numbers) makes for a clearer equation as shown!

Dropping something on surface of Earth from a height – final speed?

$v_f = ?$ $\quad v_0 = 0$ $\quad r_0 = C R_E$ (where C = some number like 3 or 4.7 or whatever) $\quad r_f = R_E$

$$0 - \frac{GM_E m}{r_0} = \frac{1}{2} m v_f^2 - \frac{GM_E m}{R_E} \Rightarrow v_f^2 = \frac{2GM_E}{R_E} - \frac{2GM_E}{r_0}$$

factor out $2GM_E$ from right $\quad v_f^2 = 2GM_E\left(\frac{1}{R_E} - \frac{1}{r_0}\right)$

combine fractions $\quad v_f^2 = 2GM_E\left(\frac{r_0 - R_E}{R_E r_0}\right)$

if $r_0 = C R_E$ $\quad v_f^2 = 2GM_E\left(\frac{(C-1)R_E}{C R_E^2}\right)$

$$v_f^2 = \frac{2GM_E}{R_E}\left(\frac{C-1}{C}\right) = v_{esc}^2\left(\frac{C-1}{C}\right) = v_f^2$$

$$v_f = v_{esc}\sqrt{\frac{C-1}{C}}$$

where $r_0 = C R_E$

see example

Suppose $h = 4R_E$

$\Rightarrow r_0 = R_E + 4R_E = 5R_E$

$\Rightarrow C = 5$

$v_f = (11200)\sqrt{\frac{4}{5}} = .89\, v_{esc}$

can work with fractions! $\quad h = 4.2 R_E \quad r_0 = 5.2 R_E$

$C = 5.2$

$$v_f = v_{esc}\sqrt{\frac{4.2}{5.2}}$$