## Twin Paradox

Suppose one twin stays on Earth and other twin goes off in space ship very fast. Each will observe the other one as aging less than them ... who is right? The twin in the ship will be younger than the twin on the Earth, because the Earth twin doesn't change inertial reference frames. Explanation follows:

## Earth View of Twin Paradox:

Suppose Planet X is 12 light years away from the Earth (fixed distance). The ship moves at 0.6 c so will take 20 years to get to the Planet and 20 years back (as seen by an observer fixed to Earth's reference frame). So, the twin on Earth will age 40 years. The ship sends a signal back toward Earth every year (frequency $=1 /$ year) - they send the signal when THEY think a year has passed! (We can later extrapolate this to a continuous "video" signal.)


But, when the ship moves away from the Earth, the frequency observed by the Earth is shifted down (and when the ship moves toward the Earth, the frequency shifts up) :

$$
f_{\text {recede }}^{\prime}=(1 / \text { year }) \sqrt{\frac{1-0.6}{1+0.6}}=(0.5 / \text { year }) \quad f_{\text {approach }}^{\prime}=(1 / \text { year }) \sqrt{\frac{1+0.6}{1-0.6}}=(2 / \text { year })
$$

So, on the way out, the Earth would only receive the "yearly" signal from the ship every two years. (Applying this analogy to a "continuous video feed" from the ship, the action on the ship would appear to be going twice as slow as it should be - slow motion). The Earth will receive 10 signals from the ship in the time it takes the ship to get to the planet. (The ship takes 20 Earth years to get there, and Earth receives 1 signal every two years ... thus 10 signals). So, to Earthbound observers of the continuous video feed from the receding ship, the "action" on the ship seems to be moving at half speed. (Since the ship has sent 16 signals by the time it reaches the planet, why did the Earth only receive 10 signals ... 6 are still in space, heading back to the planet!)


When the ship arrives at the planet, it will send back it's normal signal, but that signal will have to travel back to the Earth before the Earth has proof that the ship has landed.
(At the 20 year mark on Earth, the Earth can "assume" the ship arrived, but it won't know for certain for another 12 years, until the signal comes from the planet.) Until it receives that "arrival" signal from the planet, the Earth is still viewing the first half of the trip in slow motion (since the signals are "backed up" in space, on route to the Earth). In fact, the ship has already turned around and is heading back toward the Earth while the Earth is still seeing the outward trip. The "arrival" signal will be the last of the slow-motion signals. That will arrive at the earth at $20+12$ years $=32$ years into the trip (as seen by the Earth).


So, the ship has turned around and is heading back. At the 32 -year mark on Earth, the Earth starts receiving signals at a rate of 2 signals per year (because the signals from the returning ship start arriving in a bunch). For the last 8 Earth years of the journey, the Earth will receive 16 signals from the ship (the action of the ship has now speed up to twice normal speed). At the 40 year mark on Earth, the ship arrives, having only transmitted $16+16=32$ signals $=32$ years. So the twin on the ship will have been seen to age 32 years, while the twin on the Earth aged 40 years.

There are two parts to this ... one is the very logical time lag with the signals arriving at the Earth - that is a non-relativistic phenomenon - the fact that the ship has already turned around and is heading back before the Earth KNOWS it has arrived. The relativity comes in from the Doppler shifting of the frequency of signals (thus the observed passage of time).

## Ship's view of the Twin Paradox:

On the way out ... the ship receives signals from Earth and perceives a rate of $0.5 / \mathrm{year}$. The ship looks at the distance to the Planet X and sees it length-contracted:

$$
L^{\prime}=L_{0} \sqrt{1-(0.6)^{2}}=(12 \text { cyear }) \sqrt{0.64}=0.8(12 \text { cyear })=9.6 \text { cyear }
$$

Traveling at 0.6 c , the ship will perceive 16 years travel time to the planet. [This is the key point ... this involves the fact that the ship is moving relative to the inertial reference frame.] So, during those 16 years ... they will receive 8 signals from the Earth (Earth appears slower to the ship). Then, they will turn around and fly back; as soon as they turn around, they begin receiving signals at twice the normal speed ( 2 signals a year). It takes 16 years of ship time to go back ... that will mean 32 signals from the Earth ... so, they will perceive the $(8+32)=40$ years of the Earth time, while they will count 32 years on their own clocks.

Key difference between the two views - The Earth perceives correct distance to the Planet and counts the time to the planet correctly. The Ship perceives a different distance because it is moving relative to the space where the true distance is measured. [Sort of a cheat ... we are forcing the solution by length-shifting the Ship's view ... that's because it is in the non-inertial reference frame. But, with that reasoning accepted, the solution can be "proven" using the relativistic equations.

