Barycenter of Solar System

Earth-Moon barycenter?

Moon orbits … what?

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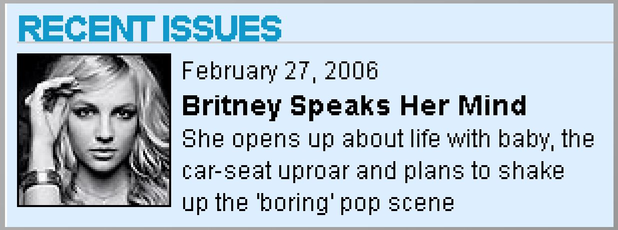
Webpage :

http://qbx6.ltu.edu/s_schneider/astro/astroweek_2006.shtml

Brought to you by …

- The Letter Q (as in Quisp)
- The number 3.1415
- Code Red!
- Diet Coke (after the sugar shakes set in)

- …. Britney Spears?



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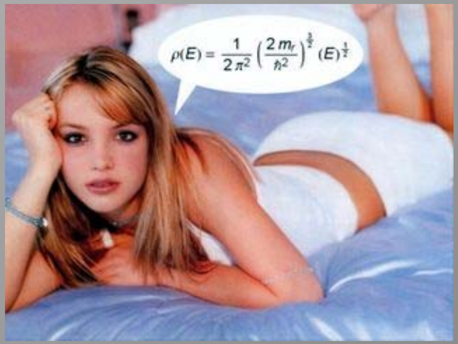


Who orbits what?

- Planets in the solar system orbit the Sun?
- Actually they orbit the “center of mass”
  - Called the “barycenter” of the solar system
- Barycenter has “smooth” path through galaxy (but Sun/planets “wiggle”)
  - Earth/Moon system also has a barycenter

How do you calculate the barycenter?

- Start with – how to you find the positions of the planets
- Fancy dancy powerful computers can calculate the positions based on Newton's law of Gravity (and Einstein (JPL)
- Those "time series" (way too large to be 'emailed') can be analyzed (VSOP theory)
- Fourier series approximation (based on time)
  - Now you just need to know the coefficients
  - Can then reconstruct the position information

To find the planet locations …

- Pick a planet .. (poor Pluto too tough, but too far away)
- Pick a date …
- Pick a coordinate system …
  - Heliocentric = measured from the Sun (center)
    - Earth-Sun plane – “point of Aries” (Earth-Sun: Spring)
  - Geocentric = measured from Earth (center)
- Get back either X, Y, Z …
  - Measured in Astronomical Units (AU) (Earth-Sun distance)
- or *l* (longitude), b (latitude), r (radial distance)

"Just" some sums … (many!)

$$L = L_0 + L_1 T + L_2 T^2 + L_3 T^3 + L_4 T^4 \quad T = \text{ time in centuries from Y2000}$$

$$L_0 = \sum A\cos(B + CT) \quad \text{tables of coeff} = 100\text{'s}, 1000\text{'s?}$$

Similar for L_1, L_2, … then for B - then R

Apply some correction terms (eccentrities, Relativity, etc.) ….

Then go on to the next planet (or time) …

- For each of the coordinates (x, y, z or l, b, r) – have up to 5 coefficients for a "time polynomial"
- Coefficients generated from long cosine series
  - Some series have over 2000 terms – for one coefficent – for one variable!

First you get the planets, then you get the power …

- If you have all the locations of the planets
  - Let's use x, y, z for the planets
- Can now find the center of mass of the system
  - "weighted average" of the planets + Sun
  - Measured from the center of the Sun
    - Since planet distances measured from Sun, we find barycenter from center of sun
- Can watch barycenter move around disk of the Sun
  - Remember, actually sun (and planets) orbit barycenter!

Equations for Barycenter

$$X_{BC} = \frac{\sum_{i=1 \text{ to } 8} m_i x_i}{M_{tot}} \qquad Y_{BC} = \frac{\sum_{i=1 \text{ to } 8} m_i y_i}{M_{tot}} \qquad Z_{BC} = \frac{\sum_{i=1 \text{ to } 8} m_i z_i}{M_{tot}}$$

$$M_{tot} = M_{sun} + M_{all\ the\ planets}$$

- What are the subscripts 1 through 8??
- Once we have X, Y, Z for barycenter – can calculate the net distance from the Sun center
  - Denote this distance as G – units of Sun radius

How to plot the barycenter info?

- Can plot distance G against time
  - Watch for highs and lows
  - When is it near Sun center – far from Sun center?
- Can plot barycenter on xy graph of Sun's disk
  - Watch path of barycenter
    - Remember – really Sun moving around barycenter
    - Get too dizzy to try to have Sun wiggle around barycenter!
- Since we know the planet locations – can also plot them on a "solar system" xy plane
  - Might be interesting to see where the planets are when the Sun is near or far from the barycenter!

Computer Program Demo!

Results from the Barycenter demo

- The barycenter not confined to the Sun’s interior!
  - From 1940-2060 – G = outside Sun 62 % of the time
  - Can also spend more than 10 years inside
- Jupiter is the “big dog” planet!
  - When G is near zero – often have Jupiter on one side – other planets on the other side
- G can be more than 2 solar radii
  - This usually means all the planets are “on the same side” of the Sun
- G = 1 passing through radius of sun
  - Watch how the shift in the planets is related to this!
- How would we use this info about a solar system barycenter?
  - A “star” would “wiggle” to an outside observer – based on the planets …

Earth/Moon barycenter

- Earth mass much larger than Moon
  - Keeps the barycenter inside the Earth
  - Distance from Earth center is 70-80% Earth radius
    - Why does the distance vary?
- Boring to plot that (basically a circle)
  - No cool looping in and out of the radius ..
- Where does the Earth-Moon line stick out?
- Can calculate location of moon
  - Similar to the planets – a “series” calculation
  - Then plot it on the surface of the Earth – longitude/latitude
  - Seems easy – how could anyone make a mistake?

Computer Program Demo!

Rookie Mistakes! … and results

- Forgot to take into account Earth axis tilt – doh!
- Forgot to take into account rotation of Earth!
- Short time (1 year) – paths of moon – shift a little
  - Why is there any shift at all in a year?
- Longer time (10 years) – starts to fill in band
  - Why does it fill in? What factors?
- What else does that band represent?
  - What if there was an observer along one of those tracks?

Moon orbits the Earth, right?

- Seems like an obvious question?
  - This was at least one thing the ancients got right!
- We have full moons and new moons!
  - Moon must get between us and the Sun!
- What does the orbit look like in space
  - Jump in a rocket and rise above the Earth-Sun plane
  - Look down on the Moon orbit as the Earth-Moon system goes around the Sun
- My prediction is that there are NO loop-de-loops!

Skygazer Demo 1!

Um … uh …. er …

- Ok, I really thought there wasn't supposed to be any outward motion of the moon …
- Wait .. What was that line about x400 scale?
- Could the scaling of the moon orbit create that motion (the motion that offends me!)?
- Let's try scaling down the moon-orbit?

Skygazer Demo 2!

Ok, well, that's better …

- Scaling the system by 400x gave loops
  - Note: MANY books show the orbit like that!
- Bringing the scale down to 100x got rid of the loops …
- But, there is still the "curving outward" part?
- Let's try scaling down the moon-orbit again?

Skygazer Demo 3!

Almost there ….

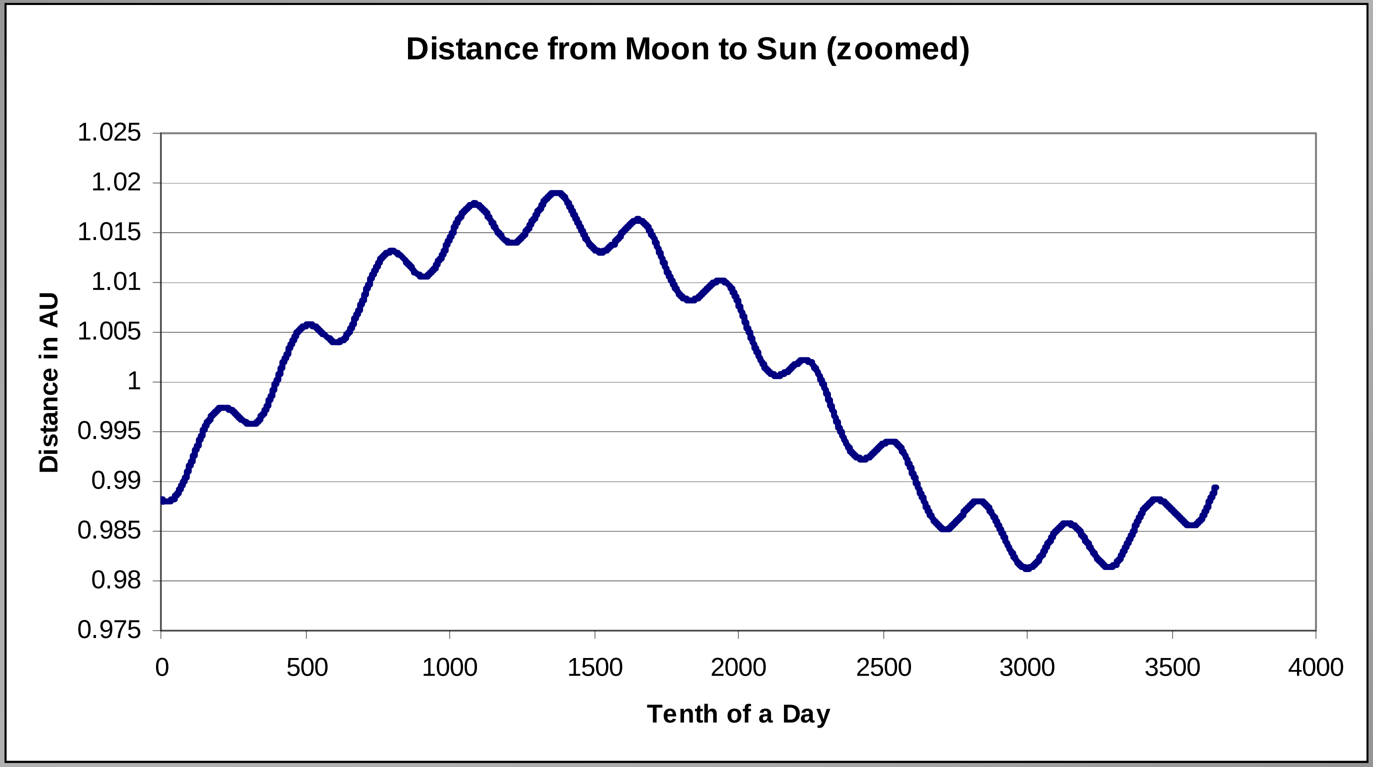
- Scaling by 20 gives us a hint that maybe the moon path doesn’t curve outward
- Run Demo 4 – 1 to 1 scale
  - Note: we can remove Earth orbit just in case
- Now it looks like the orbit is only curving inward – never bends outward
- Can we prove that with data?

Skygazer Demo 4!

Calculate Moon AND Earth!

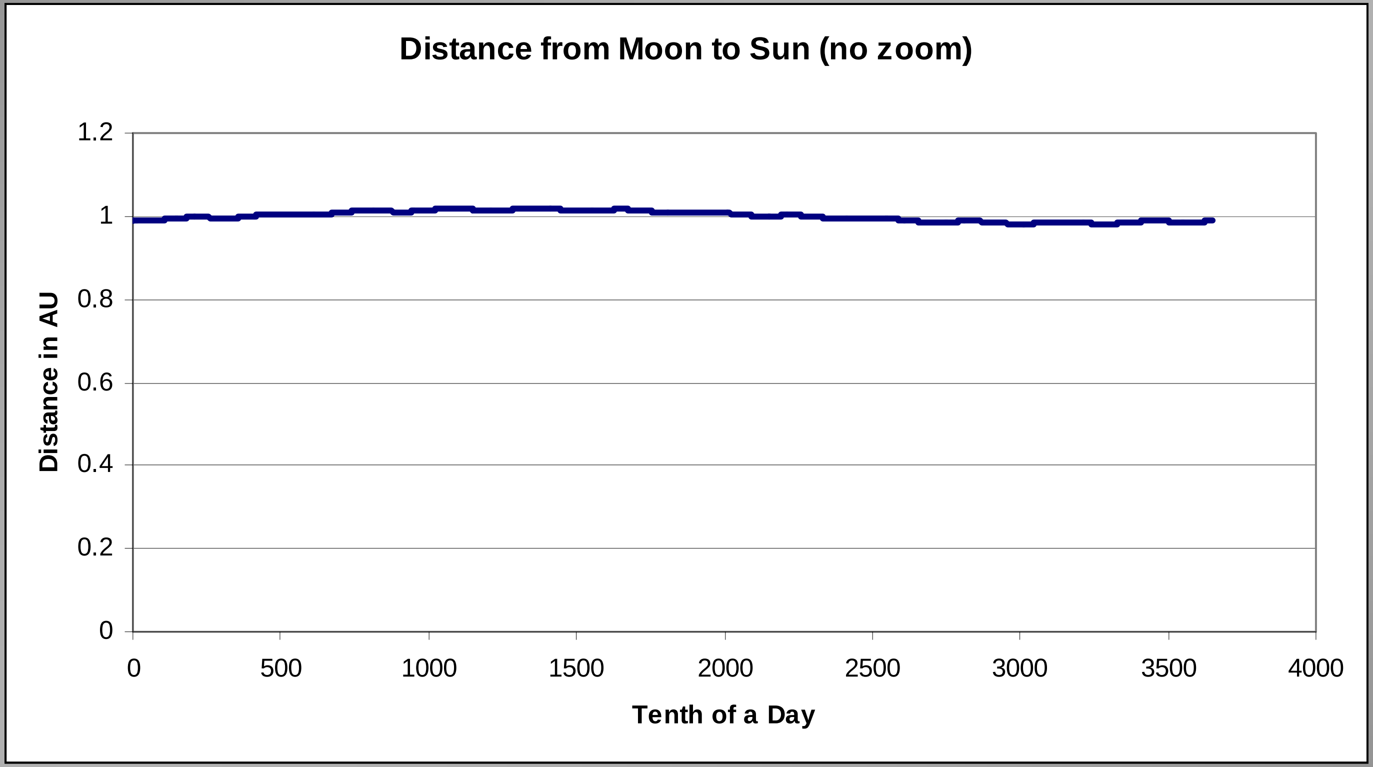
- Find the distance of the Earth to the Sun
- Find the distance from Moon to Earth
- Add them together – find the Moon-Sun distance!
  - Should be able to show that it doesn't curve outward!
- So .. On to the results!

Umm … uhh … errr …



- Wait … maybe if I just change the vertical scale?

Umm … uhh … errr … part 2



- Any ideas? (Not Brad!)

Can’t plot on XY axes!

- The “curving outward” is a “polar plot” idea
- Clearly the Moon has to occasionally come closer to the Sun than the Earth, and go farther out
  - If “curving inward” meant getting closer to the Sun all the time – it would crash!
- Then what does it mean to be always curving inward?
  - Here is an example … an ellipse!
    - Clearly the object in elliptical orbit comes closer and farther
    - But, the orbit path never curves outward!
  - Very clever – but the Moon orbit isn’t exactly elliptical

How can we show the curvature?

- Dr. Scott puzzled and puzzed, until his puzzler was sore!
  - Note: Dr. Scott never re-uses jokes – just ask any of the students that have him for more than one term!

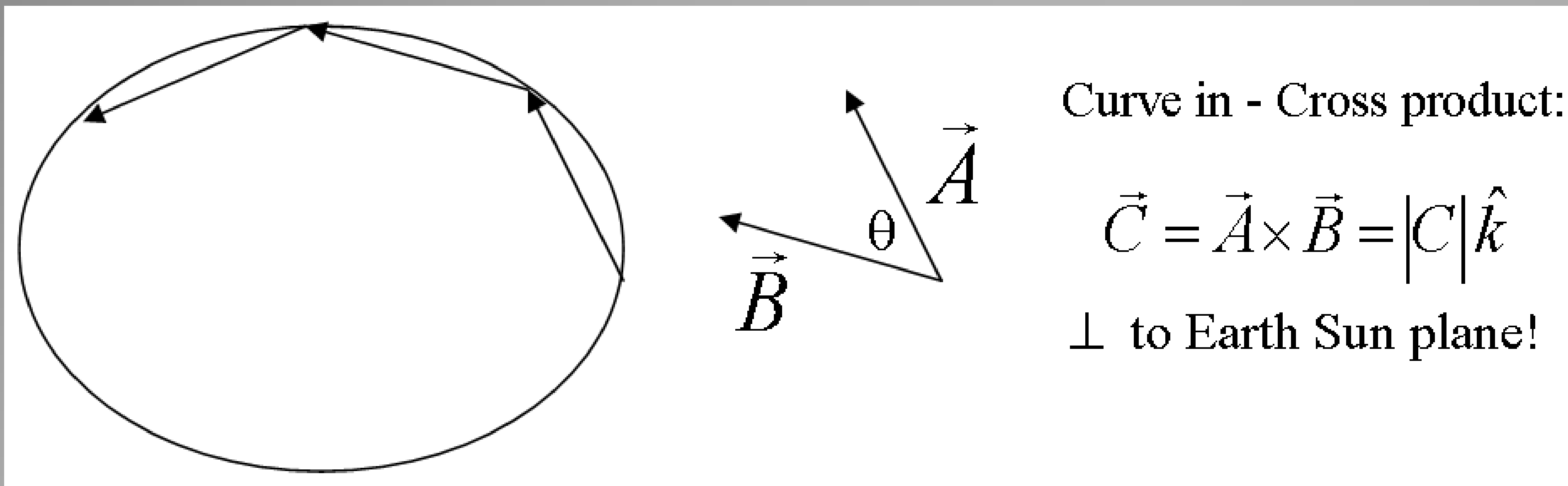
Calculus to the rescue!!

- A day spent **not** thinking about vectors is a day wasted!
- We can use vectors to verify the “only curving inward” nature
  - And vectors very happily don’t worry about limitations like XY plots to try to illustrate curvature!
  - And, since we have already calculated the XY positions, we can easily calculate vectors
    - … and use **certain properties** of vectors (mysterious!)

What vectors are needed?

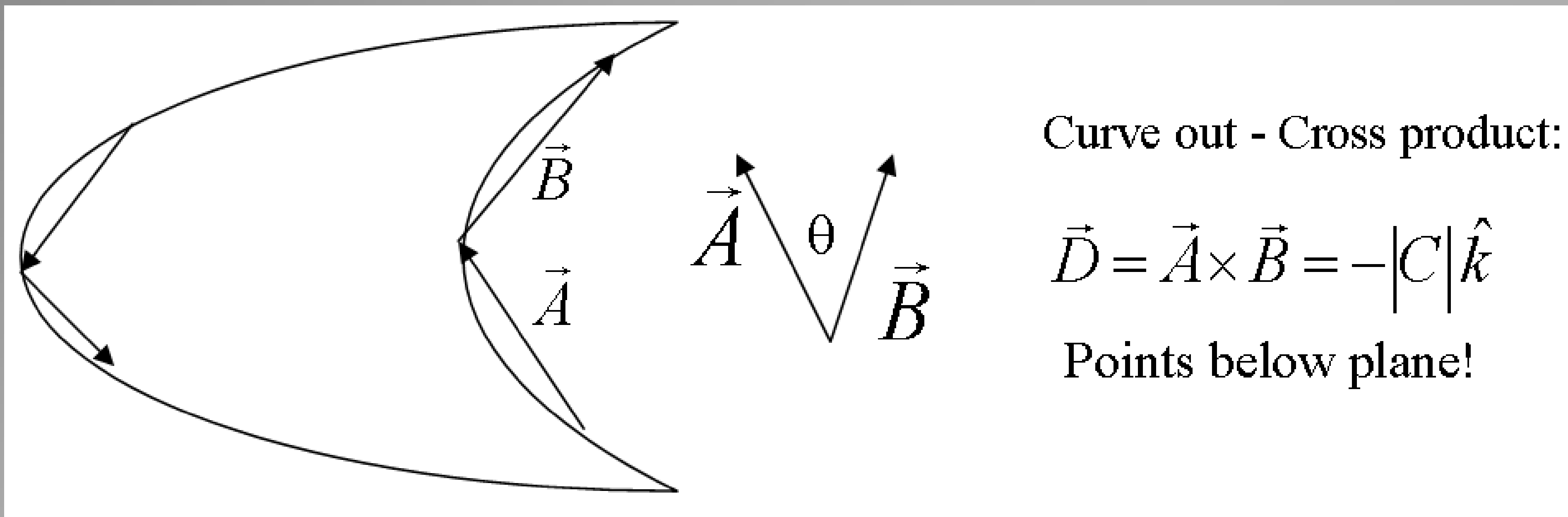
- If we connect the moon’s position on one date, to the position on the next date – we have a vector!
- And if we join the second position to a third – another vector
- Those two vectors will have a relative angle between them
- Can create a “**cross product”** of the vectors
  - If the two vectors are only the XY plane, the cross product will be in the Z direction only!!
  - And, +z or –z direction determined by curvature!

Simple ellipse with vectors and cross



- If vectors “turn inward” then cross product points out of the page/board (+z direction = +k)

Inward curve gives negative value!



- If vectors “turn outward” then cross product points **INTO** of the page/board (-z direction = -k) !!
  - We now have a method to prove the “curving inward” Moon orbit!

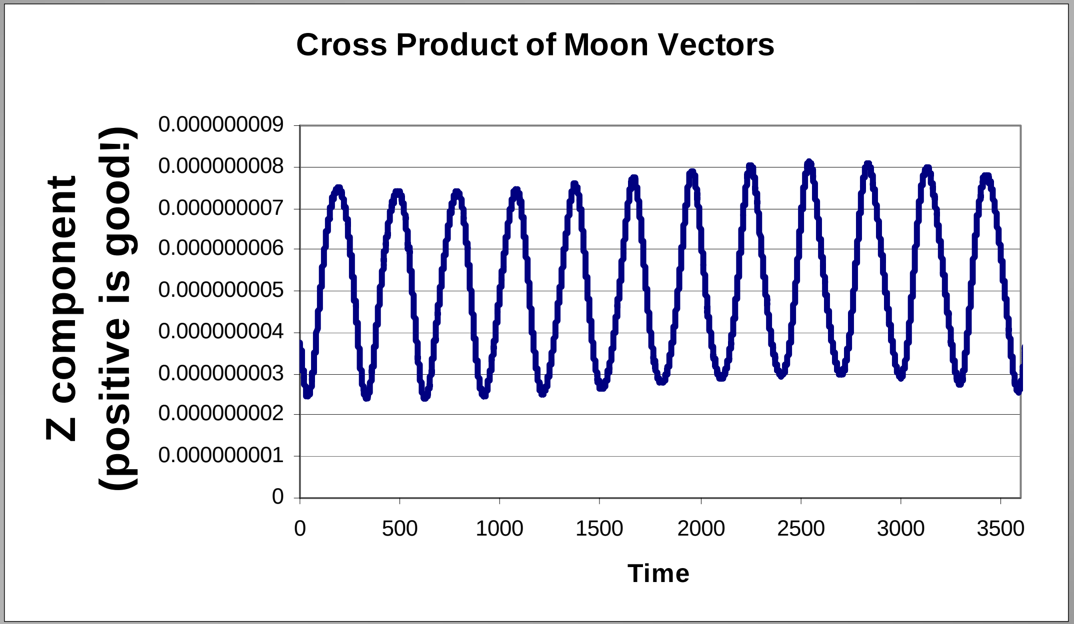
Drum roll …

$$\vec{C} = \vec{A} \times \vec{B} = (A_x B_y - A_y B_x)\hat{k}$$

$$A_x = x_2 - x_1 \quad A_y = y_2 - y_1$$

$$B_x = x_3 - x_2 \quad B_y = y_3 - y_2$$

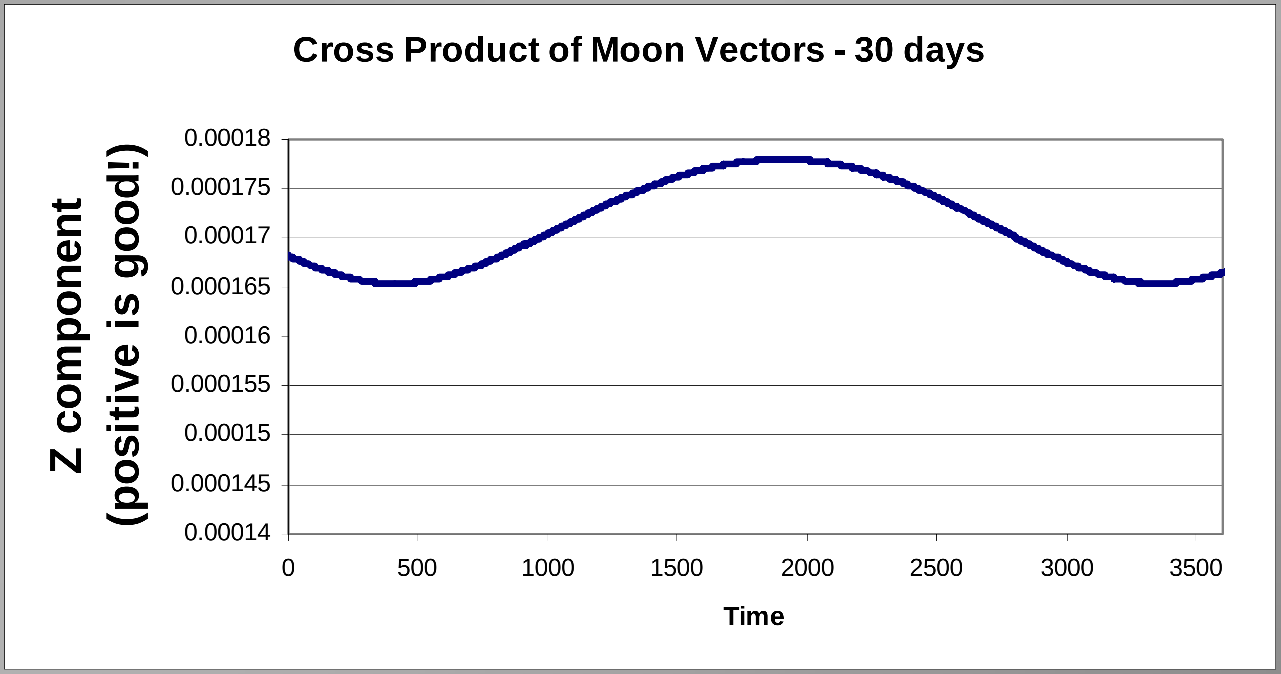
$$C_{test} = (A_x B_y - A_y B_x) > 0 \ ?$$



- And, he sticks the landing!
  - Yup, Olympics were on TV when I was doing this …
- Also note – only the X and Y values contribute to the Z component of the cross (so only using X and Y is not a problem!)

What further tests?

- More resolution - try 30 days (that would be enough for moon to go “out” then “in” – full orbit of Earth)
  - Using about 4000 points – 10x resolution! Still good!!



So what affects orbit curvature?

- Our Moon is essentially unique in the solar system
- Most other moons have the loop-de-loop type behavior
  - Some just curve outward – some loop – some do both!
- Curve-outward orbits related to mass ratios of Sun and planet – compared to the distance ratio of Moon to planet
- We can create a ‘simulation’ system – with “fake” orbits
  - The “principles” would still apply
  - Can adjust radii and orbit periods – watch cross product!

Fake "moon orbits sun" model

- Have circular orbit for planet around sun
- Have circular orbit of moon around planet
- Adjust the ratio of planet orbit period to moon orbit period
- Adjust the distance of moon-to-planet vs planet-to-sun
- Watch the cross product to see if it goes negative (loops or curves outward)!

Excel Demo!

Bonus: Planets lining up in space ?

- Planets can’t line up in a straight line (from the Sun)
  - Orbits are tilted – nodes don’t line up – periods don’t line up
  - Didn’t you see the talk last Tuesday?
- They can line up in *some other line* in space
  - We’ll consider three planets – middle occults outside ones
- One planet crosses in front of another – “occultation”
- Does each outside planet see the same event?
  - Depends on how far away they are from each other
  - Marvin the Martian and Earth and Venus – yes
  - Non-reciprocal Occultation of Jupiter Venus Mars (1930) -No

Skygazer Demos!

References

Jean Meeus is an Astronomer from Belgium – does some absolutely amazing astronomical calculations. Has many excellent books :

Astronomical Algorithms

Astronomy Morsels

More Astronomy Morsels

Astronomy Morsels III

… please .. MORE!!

This talk, and other exciting things can be found …

http://qbx6.ltu.edu/s_schneider/astro/index.shtml

Thank you,
Good Night,
Drive Safely!