Barycenter of Solar System
Moon orbits … what?

Dr. Scott Schneider

Friday Feb 24th, 2006

Sponsored by the

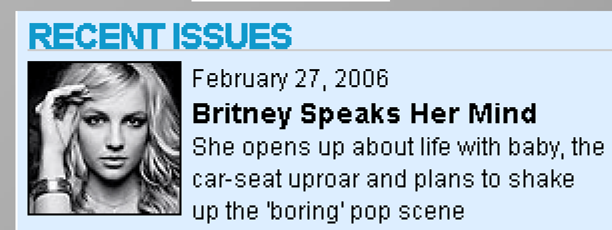
Society of Physics Students (SPS)

Webpage :
http://qbx6.ltu.edu/s_schneider/astro/astroweek_2006.shtml

Brought to you by …

- The Letter Q (as in Quisp)
- The number 3.1415
- Code Red!
- Diet Coke (after the sugar shakes set in)

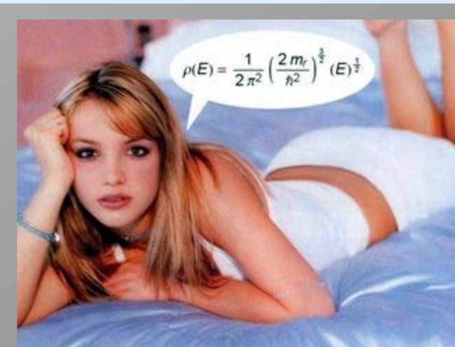
- …. Britney Spears?


RECENT ISSUES

February 27, 2006

Britney Speaks Her Mind

She opens up about life with baby, the car-seat uproar and plans to shake up the 'boring' pop scene



Who orbits what?

- Planets in the solar system orbit the Sun?
- Actually they orbit the "center of mass"
  - Called the "barycenter" of the solar system
- Barycenter has "smooth" path through galaxy (but Sun/planets "wiggle")
  - Earth/Moon system also has a barycenter

How do you calculate the barycenter?

- Start with – how to you find the positions of the planets
- Fancy dancy powerful computers can calculate the positions based on Newton's law of Gravity (and Einstein (JPL)
- Those "time series" (way too large to be 'emailed') can be analyzed (VSOP theory)
- Fourier series approximation (based on time)
  - Now you just need to know the coefficients
  - Can then reconstruct the position information

To find the planet locations …

- Pick a planet .. (poor Pluto too tough, but too far away)
- Pick a date …
- Pick a coordinate system …
  - Heliocentric = measured from the Sun (center)
    - Earth-Sun plane – "point of Aries" (Earth-Sun: Spring)
  - Geocentric = measured from Earth (center)
- Get back either X, Y, Z …
  - Measured in Astronomical Units (AU) (Earth-Sun distance)
- or *l* (longitude), b (latitude), r (radial distance)

"Just" some sums … (many!)

$L = L_0 + L_1 T + L_2 T^2 + L_3 T^3 + L_4 T^4 \quad T =$ time in centuries from Y2000

$L_0 = \sum A\cos(B + CT)$ tables of coeff = 100's, 1000's?

Similar for L_1, L_2, ... then for B - then R

Apply some correction terms (eccentrities, Relativity, etc.) ....

Then go on to the next planet (or time) ...

- For each of the coordinates (x, y, z or l, b, r) – have up to 5 coefficients for a "time polynomial"
- Coefficients generated from long cosine series
  - Some series have over 2000 terms – for one coefficent – for one variable!

First you get the planets, then you get the power …

- If you have all the locations of the planets
  - Let’s use x, y, z for the planets
- Can now find the center of mass of the system
  - “weighted average” of the planets + Sun
  - Measured from the center of the Sun
    - Since planet distances measured from Sun, we find barycenter from center of sun
- Can watch barycenter move around disk of the Sun
  - Remember, actually sun (and planets) orbit barycenter!

Equations for Barycenter

$$X_{BC} = \frac{\sum_{i=1 \text{ to } 8} m_i x_i}{M_{tot}} \qquad Y_{BC} = \frac{\sum_{i=1 \text{ to } 8} m_i y_i}{M_{tot}} \qquad Z_{BC} = \frac{\sum_{i=1 \text{ to } 8} m_i z_i}{M_{tot}}$$

$$M_{tot} = M_{sun} + M_{all\ the\ planets}$$

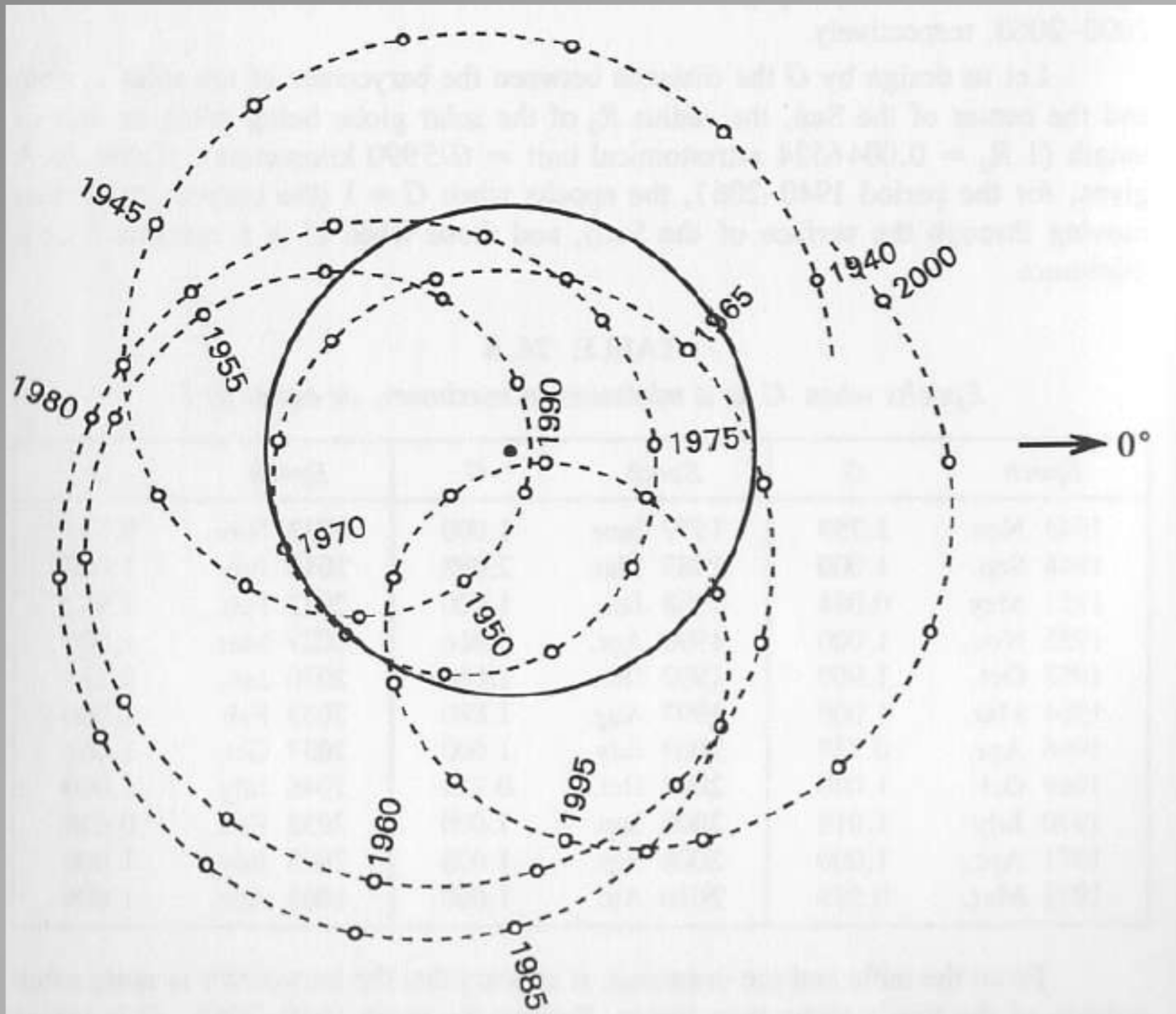
- What are the subscripts 1 through 8??
- Once we have X, Y, Z for barycenter – can calculate the net distance from the Sun center
  - Denote this distance as G – units of Sun radius

How to plot the barycenter info?

- Can plot distance G against time
  - Watch for highs and lows
  - When is it near Sun center – far from Sun center?
- Can plot barycenter on xy graph of Sun's disk
  - Watch path of barycenter
    - Remember – really Sun moving around barycenter
    - Get too dizzy to try to have Sun wiggle around barycenter!
- Since we know the planet locations – can also plot them on a "solar system" xy plane
  - Might be interesting to see where the planets are when the Sun is near or far from the barycenter!

Computer Program Demo!

BARYCENTER



1945
1940
2000
1965
1955
1980
1990
1975
0°
1970
1950
1960
1995
1985

Results from the Barycenter demo

- The barycenter not confined to the Sun's interior!
  - From 1940-2060 – G = outside Sun 62 % of the time
  - Can also spend more than 10 years inside
- Jupiter is the "big dog" planet!
  - When G is near zero – often have Jupiter on one side – other planets on the other side
- G can be more than 2 solar radii
  - This usually means all the planets are "on the same side" of the Sun
- G = 1 passing through radius of sun
  - Watch how the shift in the planets is related to this!
- How would we use this info about a solar system barycenter?
  - A "star" would "wiggle" to an outside observer – based on the planets …

Earth/Moon barycenter

- Earth mass much larger than Moon
  - Keeps the barycenter inside the Earth
  - Distance from Earth center is 70-80% Earth radius
    - Why does the distance vary?
- Boring to plot that (basically a circle)
  - No cool looping in and out of the radius ..
- Where does the Earth-Moon line stick out?
- Can calculate location of moon
  - Similar to the planets – a “series” calculation
  - Then plot it on the surface of the Earth – longitude/latitude
  - Seems easy – how could anyone make a mistake?

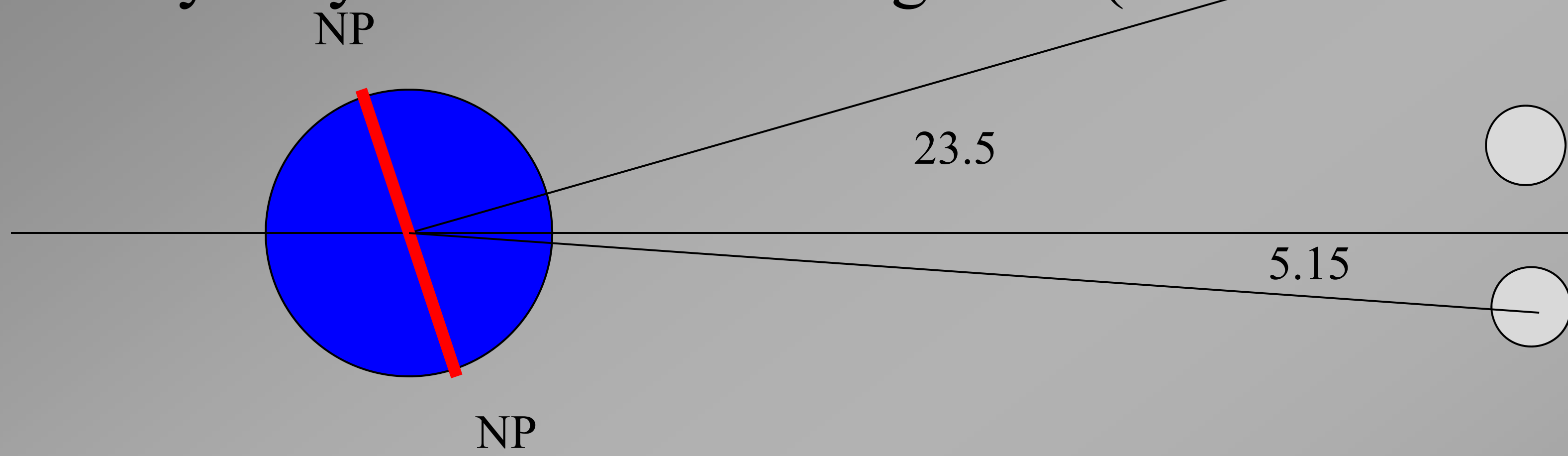
Computer Program Demo!

Rookie Mistakes! … and results

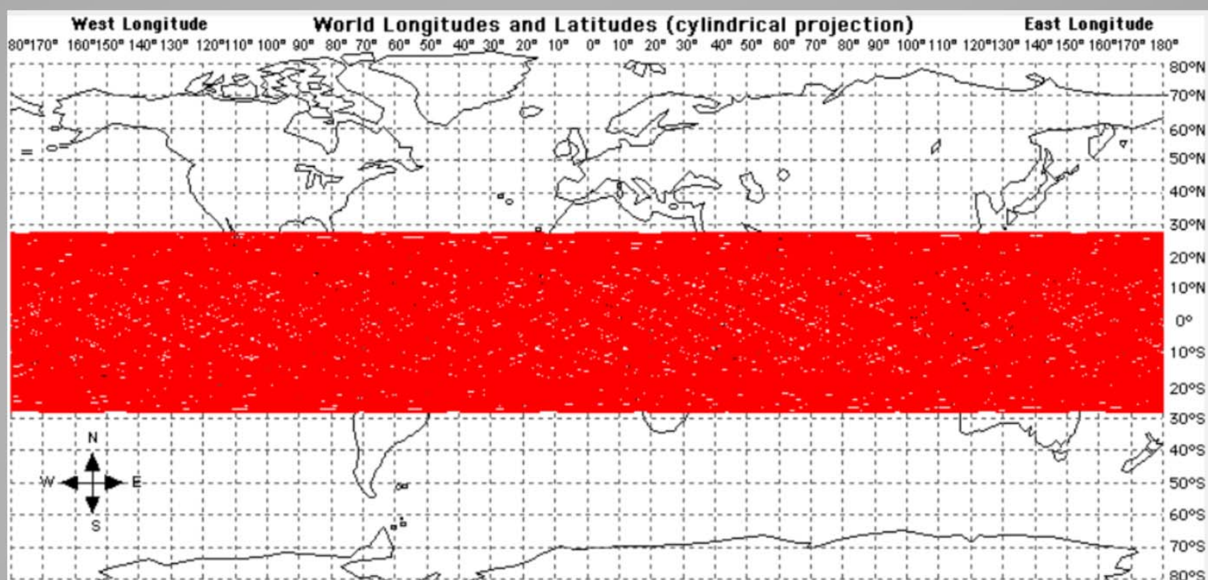
- Forgot to take into account Earth axis tilt – doh! (23.5^0)
- Forgot to take into account rotation of Earth!
- Why only about +/- 23 degrees? (Moon orbit +/- 5^0)
- Short time (1 year) – paths of moon – shift a little
  - Why is there any shift at all in a year?
- Longer time (10 years) – starts to fill in band
  - Why does it fill in? What factors?
- What else does that band represent?
  - What if there was an observer along one of those tracks?

Path correct?

- Why only about +/- 23.5 degrees? (Moon orbit +/- 5.15^0)



- So, maximum latitude angle of band ~ 28.7 deg



Moon orbits the Earth, right?

- Seems like an obvious question?
  - This was at least one thing the ancients got right!
- We have full moons and new moons!
  - Moon must get between us and the Sun!
- What does the orbit look like in space
  - Jump in a rocket and rise above the Earth-Sun plane
  - Look down on the Moon orbit as the Earth-Moon system goes around the Sun
- My prediction is that there are NO loop-de-loops!

Um … uh …. er …

- Ok, I really thought there wasn't supposed to be any outward motion of the moon …
- Wait .. What was that line about x400 scale?
- Could the scaling of the moon orbit create that motion (the motion that offends me!)?
- Let's try scaling down the moon-orbit?

Ok, well, that’s better …

- Scaling the system by 400x gave loops
  - Note: MANY books show the orbit like that!
- Bringing the scale down to 100x got rid of the loops …
- But, there is still the “curving outward” part?
- Let’s try scaling down the moon-orbit again?

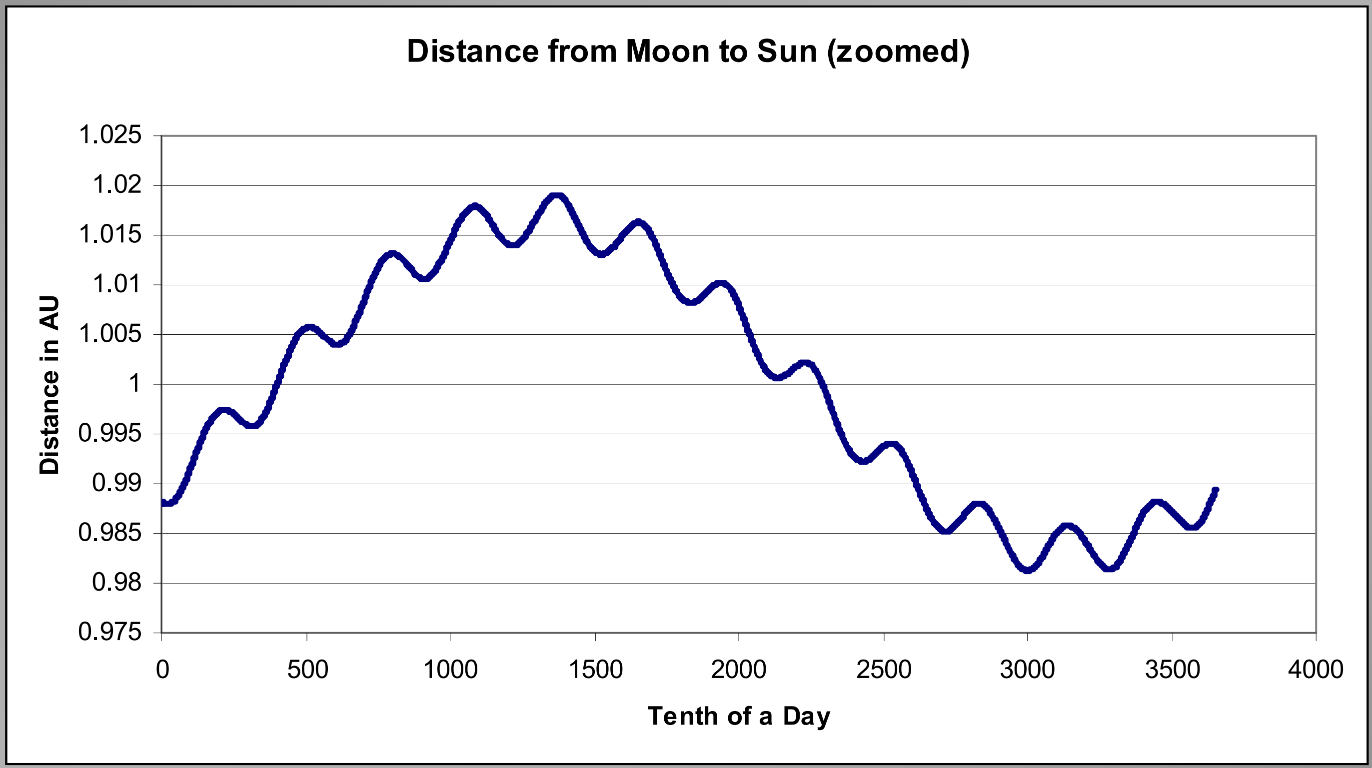
Almost there ….

- Scaling by 20 gives us a hint that maybe the moon path doesn't curve outward
- Run Demo 4 – 1 to 1 scale
  - Note: we can remove Earth orbit just in case
- Now it looks like the orbit is only curving inward – never bends outward
- Can we prove that with data?

Calculate Moon AND Earth!

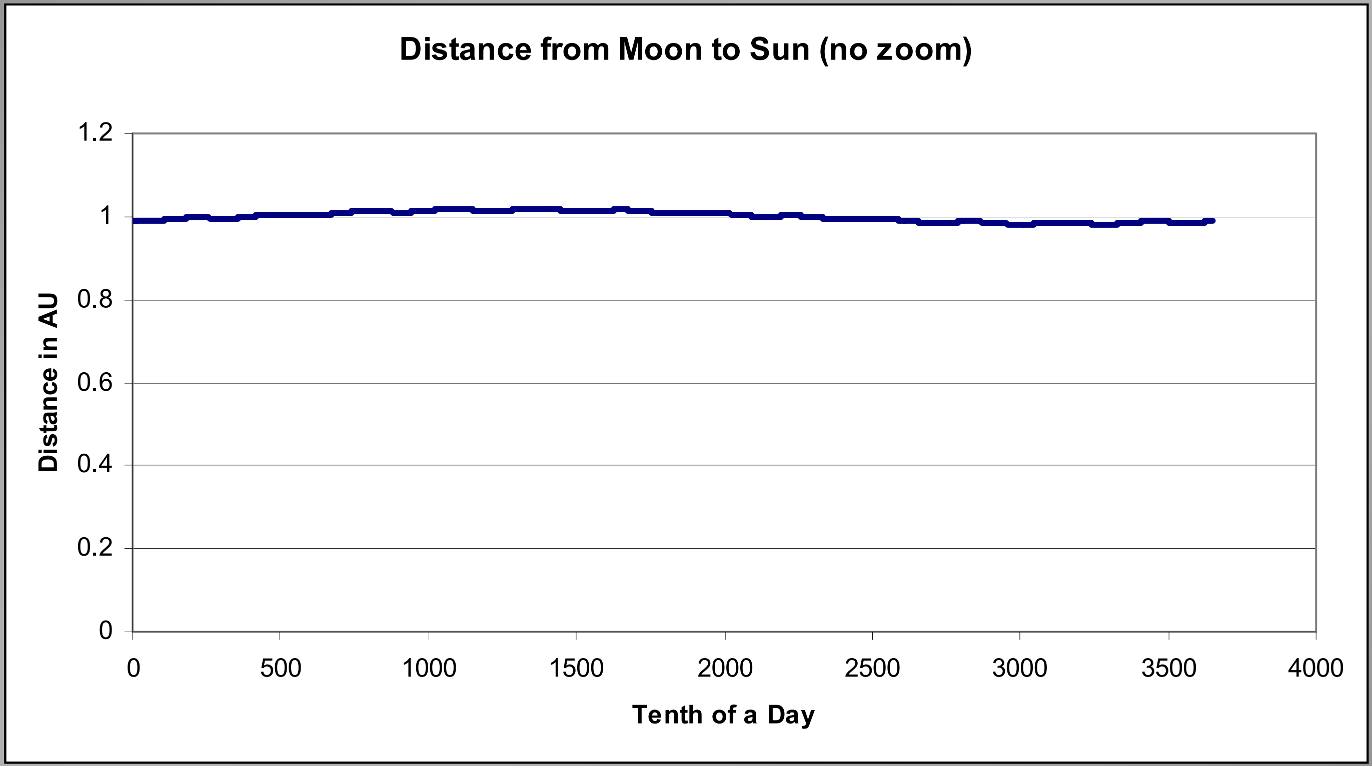
- Find the distance of the Earth to the Sun
- Find the distance from Moon to Earth
- Add them together – find the Moon-Sun distance!
  - Should be able to show that it doesn't curve outward!
- So .. On to the results!

Umm … uhh … errr …



- Wait … maybe if I just change the vertical scale?

Umm … uhh … errr … part 2



- Any ideas?

Can’t plot on XY axes!

- The “curving outward” is a “polar plot” idea
- Clearly the Moon has to occasionally come closer to the Sun than the Earth, and go farther out
  - If “curving inward” meant getting closer to the Sun all the time – it would crash!
- Then what does it mean to be always curving inward?
  - Here is an example … an ellipse!
    - Clearly the object in elliptical orbit comes closer and farther
    - But, the orbit path never curves outward!
  - Very clever – but the Moon orbit isn’t exactly elliptical

How can we show the curvature?

- Dr. Scott puzzled and puzzed, until his puzzler was sore!
  - Note: Dr. Scott never re-uses jokes – just ask any of the students that have him for more than one term!

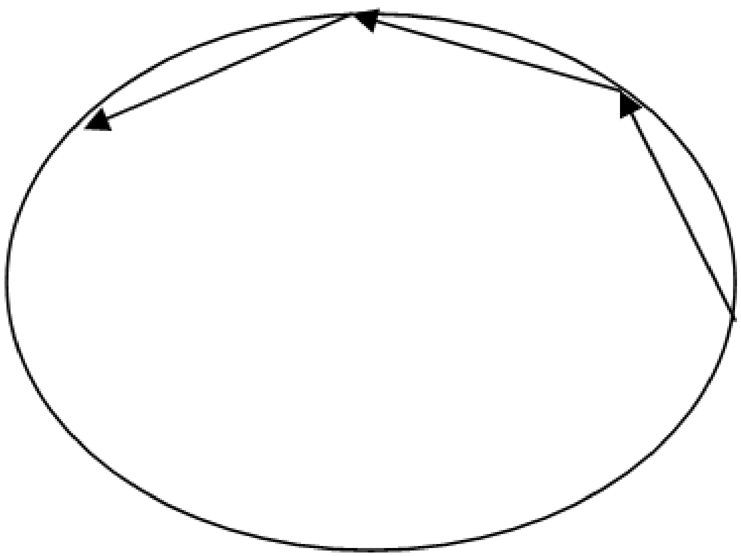
Calculus to the rescue!!

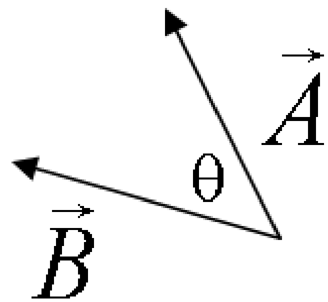
- A day spent **not** thinking about vectors is a day wasted!
- We can use vectors to verify the “only curving inward” nature
  - And vectors very happily don’t worry about limitations like XY plots to try to illustrate curvature!
  - And, since we have already calculated the XY positions, we can easily calculate vectors
    - … and use **certain properties** of vectors (mysterious!)

What vectors are needed?

- If we connect the moon’s position on one date, to the position on the next date – we have a vector!
- And if we join the second position to a third – another vector
- Those two vectors will have a relative angle between them
- Can create a “**cross product”** of the vectors
  - If the two vectors are only the XY plane, the cross product will be in the Z direction only!!
  - And, +z or –z direction determined by curvature!

Simple ellipse with vectors and cross





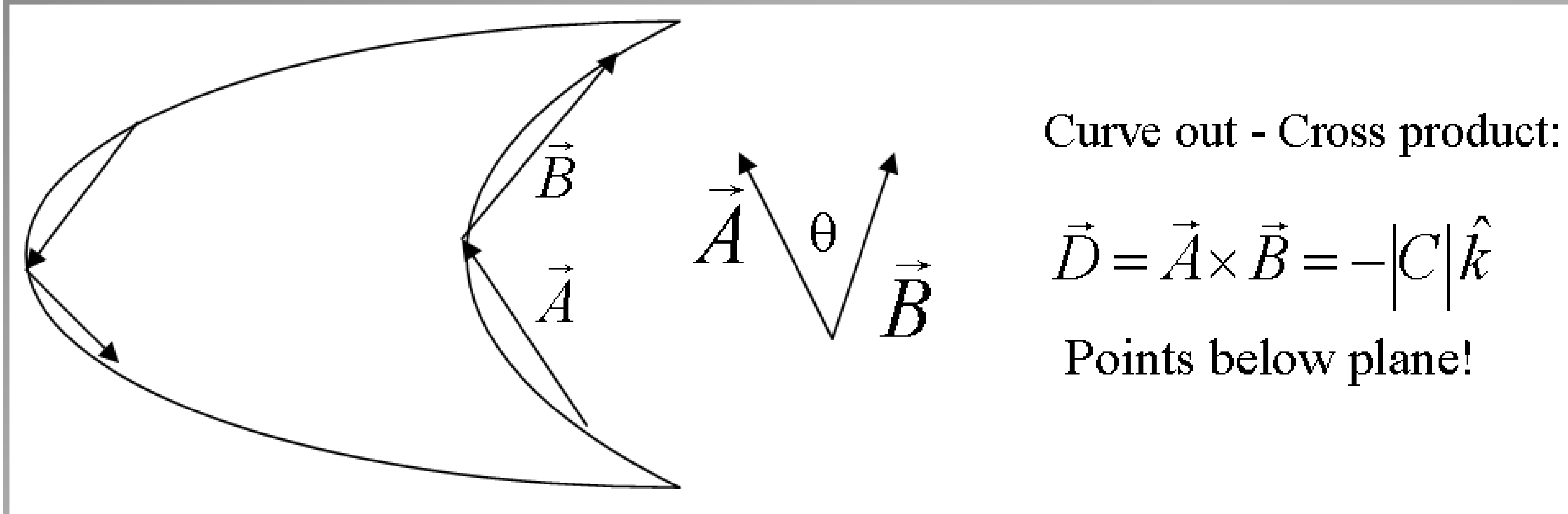
Curve in - Cross product:

$$\vec{C} = \vec{A} \times \vec{B} = |C|\hat{k}$$

⊥ to Earth Sun plane!

- If vectors "turn inward" then cross product points out of the page/board (+z direction = +k)

Inward curve gives negative value!



- If vectors "turn outward" then cross product points **INTO** of the page/board (-z direction = -k) !!
  - We now have a method to prove the "curving inward" Moon orbit!

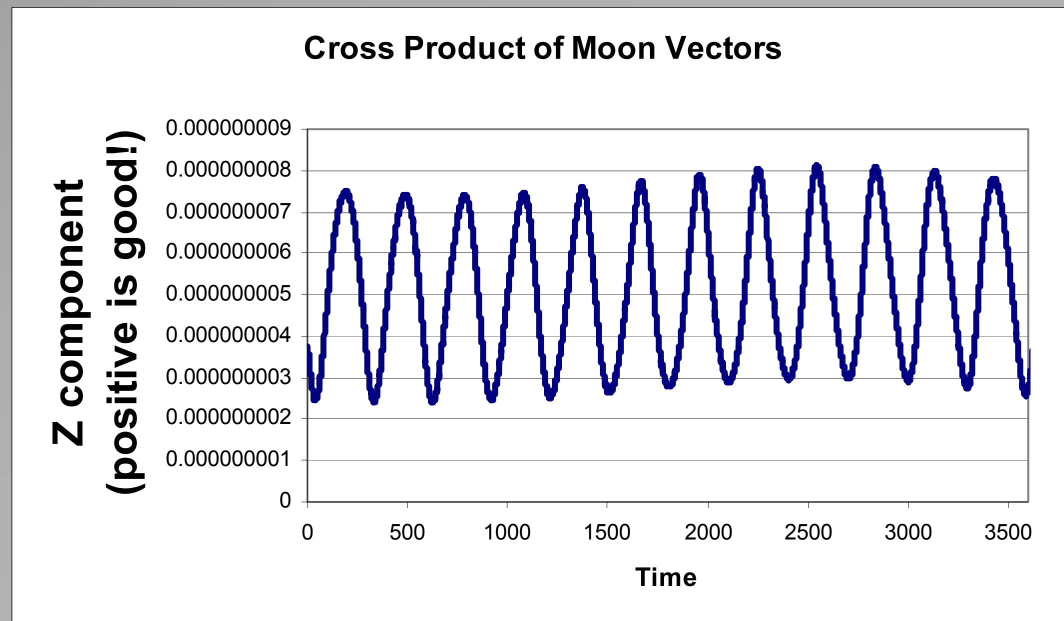
Drum roll …

$$\vec{C} = \vec{A} \times \vec{B} = (A_x B_y - A_y B_x)\hat{k}$$

$$A_x = x_2 - x_1 \quad A_y = y_2 - y_1$$

$$B_x = x_3 - x_2 \quad B_y = y_3 - y_2$$

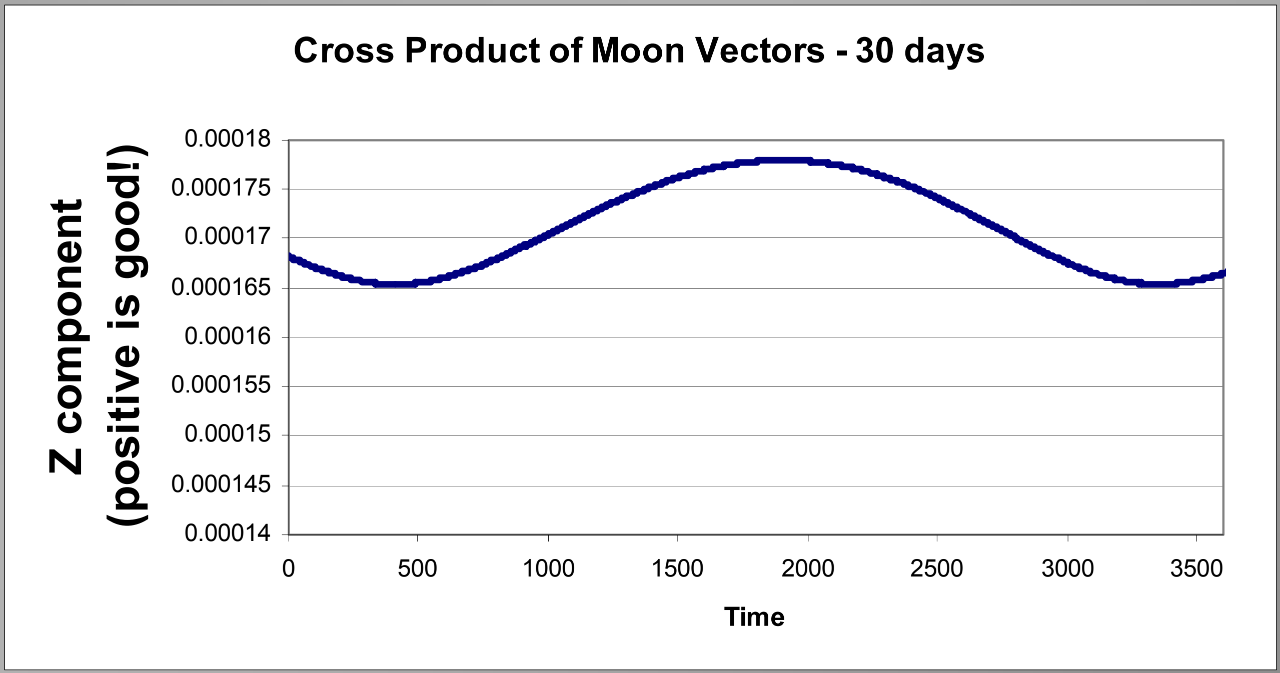
$$C_{test} = (A_x B_y - A_y B_x) > 0 \ \ ?$$



- And, he sticks the landing!
  - Yup, Olympics were on TV when I was doing this …
- Also note – only the X and Y values contribute to the Z component of the cross (so only using X and Y is not a problem!)

What further tests?

- More resolution - try 30 days (that would be enough for moon to go “out” then “in” – full orbit of Earth)
  - Using about 4000 points – 10x resolution! Still good!!



So what affects orbit curvature?

- Our Moon is essentially unique in the solar system
- Most other moons have the loop-de-loop type behavior
  - Some just curve outward – some loop – some do both!
- Curve-outward orbits related to mass ratios of Sun and planet – compared to the distance ratio of Moon to planet
- We can create a ‘simulation’ system – with “fake” orbits
  - The “principles” would still apply
  - Can adjust radii and orbit periods – watch cross product!

Fake “moon orbits sun” model

- Have circular orbit for planet around sun
- Have circular orbit of moon around planet
- Adjust the ratio of planet orbit period to moon orbit period
- Adjust the distance of moon-to-planet vs planet-to-sun
- Watch the cross product to see if it goes negative (loops or curves outward)!

Excel Demo!

References

Jean Meeus is an Astronomer from Belgium – does some absolutely amazing astronomical calculations. Has many excellent books :

Astronomical Algorithms

Astronomy Morsels

More Astronomy Morsels

Astronomy Morsels III

… please .. MORE!!

This talk, and other exciting things can be found …

http://qbx6.ltu.edu/s_schneider/astro/index.shtml

Thank you,
Good Night,
Drive Safely!