LAB 10 - MOMENT OF INERTIA (TPL1)

Objectives

- **To calculate the moment of inertia from the mass and the geometric dimensions of a system.**
- **To measure the moment of inertia of a system with the dynamic method.**

Preliminary activities

1. Prepare equipment. Make sure the rotary sensor is connected to the digital adapter (yellow plug →1, black plug → 2), and through USBLink to computer. OPEN the online help page for this lab. Start DataStudio.

2. Load the experiment file. Download the experiment file **Lab10_Inertia.DS** from the lab schedule webpage, and load it in DataStudio.

Part 1 Introduction to Rotational Inertia

Introduction You have previously done a lab that involved balanced torques being applied to a bar that is in rotational (and translational) equilibrium. If an unbalanced torque (a net torque) is applied to an object, there will be an overall rotation. The rotation will have an angular acceleration (α) that is proportional to the applied torque (τ), and inversely proportional to the moment of inertia of the object (I) as shown below:

$$\tau = I\alpha$$

You can apply a torque to a rotating turntable by applying tension to a string wound around an axis of the turntable. Hanging a mass from the string, over a pulley, creates the tension. The mass is allowed to drop, vertically, and thus accelerates. The linear acceleration of the mass is the same as the linear acceleration of the string, and thus of a point on the axis of the turntable. That linear acceleration of the mass is related to the angular acceleration of the apparatus by a factor of the radius of the axis cylinder. This is called a non-slip condition. You can measure the angular acceleration of the rotating platform using the rotary sensor. See the diagram and equations below:

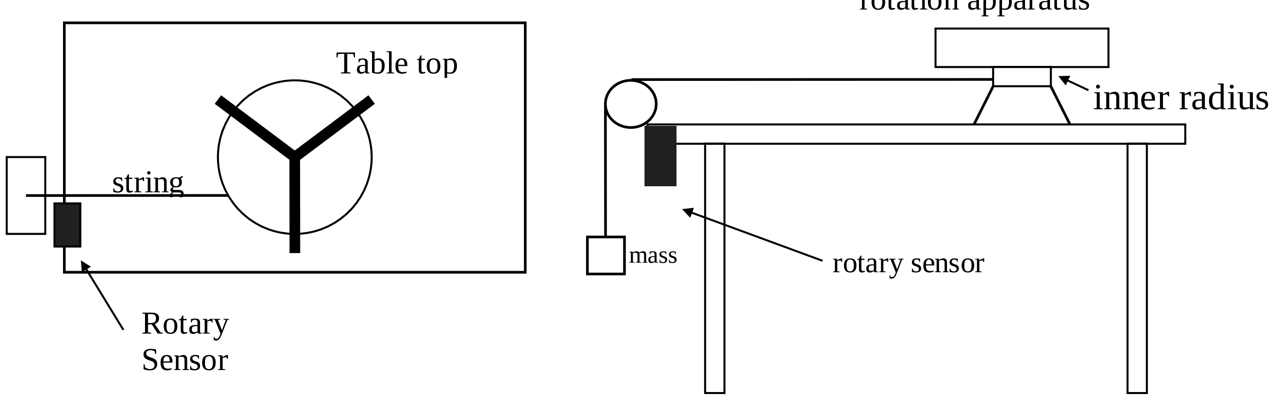

Figure 1 - Top view and side view of rotational inertia apparatus

1. Dynamics calculations. The tension in the string is dependent on the linear acceleration of the mass. That linear acceleration is related to the angular acceleration of the rotary sensor, by the radius of the rotary sensor. You can use DataStudio to measure the angular acceleration of the rotary sensor. Thus,

you can generate an expression for the tension in the string dependent on the angular acceleration of the rotary sensor.

The sum of forces on falling mass is given by: the downward force= mg, and upward force = T

$$\sum F = m_{hanging\_mass} a = m_{hanging\_mass} g - T$$
$$T = m_{hanging\_mass}(g-a) = m_{hanging\_mass}(g - \alpha_{rotary} r_{rotary})$$

The tension in the string is applied to the rotation apparatus at the distance of the inner radius (the radius of the cylinder around which the string is wrapped). This tension creates an unbalanced torque that gives an angular acceleration to the rotation apparatus. The angular acceleration is related to the moment of inertia of whatever is on the apparatus (you will put hoops and disks on the apparatus to create different moments of inertia). That angular acceleration is also related to the linear acceleration of the falling mass by the radius of the inner cylinder. Putting these relationships together (whew!), you can solve for the moment of inertia:

$$\sum \tau = I\alpha \qquad \Rightarrow \qquad I = \frac{\tau_T}{\alpha} = \frac{(T)(r)}{\alpha} = \frac{m_{hanging\_mass}(g - \alpha_{rotary} r_{rotary}) r_{apparatus}}{\alpha_{device}}$$

$$\Rightarrow \qquad I = \frac{m_{hanging\_mass} g r_{apparatus}}{\alpha_{device}} - m_{hanging\_mass} r_{rotary} r_{apparatus} \left(\frac{\alpha_{rotary}}{\alpha_{device}} \right)$$

Note that $\frac{\alpha_{rotary}}{\alpha_{device}} = \frac{r_{apparatus}}{r_{rotary}}$ … thus $\boxed{I = \frac{m_{hanging\_mass} g \left(r_{apparatus}^2 / r_{rotary} \right)}{\alpha_{rotary}} - m_{hanging\_mass} \left(r_{apparatus} \right)^2}$

[Notice that, for a given hanging mass, the numerator of the first expression and the entire second expression are constant. For each object you rotate (hoop, disk, etc.) you just need to measure the new angular acceleration from the rotary sensor and you can calculate the moment of inertia from the above equation.]

There are two objects you will put on the rotational apparatus, in various combinations: a flat disk and a hoop. To check the moment of inertia calculation above (from dynamics variables), you can use the geometric definitions of the moments of inertia for these two objects:

$$\boxed{I_{disk} = \frac{1}{2} M_{disk} R_{disk}^2 \qquad I_{hoop} = \frac{1}{2} M_{hoop} \left(R_{outside}^2 + R_{inside}^2 \right)}$$ (geometric formulas)

[NOTE: These "big R" radii are the radii for the disk and the hoop - do not confuse these with the "small r" radii of the rotary sensor or the apparatus itself.]

2. Initial measurements. To be able to use the above equations, you need to measure the inner radius of the rotation apparatus and of the rotary sensor (using a digital caliper), and the dimensions/masses of the hoops and rings. The masses of the hoops/disks are listed on the Data/Question sheets. Use a meter stick for the radius measurements on the ring and hoop. Record the values on the Data/Question sheet.

3. Preparation calculations. Using the values from Part 1.2, and the mass of the hanging mass (to start with, you will use 350 grams or 0.35 kg), calculate the two "constants" from the moment of inertia calculation above, and record them on the Data/Question sheets.

Part 2 - Dynamic Measurement of Moment of Inertia

1. Prepare equipment. Make sure that the string is wrapped around the cylinder of the rotation apparatus, and drapes over the rotary sensor (**using the OUTER pulley of the rotary sensor**). Attach 350 grams of mass to the end of the string, and "wind up" the rotation apparatus, so that the mass starts up near the table edge. When you do this, refer to the pictures in the online help page, and make sure that:

    a. The string from the rotation apparatus to the pulley is **horizontal.**
    b. The string goes over the **largest** (outer) pulley on the rotary sensor.
    c. The pulley on the rotary sensor is rotating **clockwise.**
    d. The string **lines up** with the pulley groove.

2. Measuring the moment of inertia of the rotation apparatus alone. It would be very difficult to calculate the moment of inertia of the empty apparatus geometrically, so instead you will measure it dynamically. [Later, you will subtract that value from the moments you measure with the ring and the hoop to get just the individual moments of inertia of the ring and the hoop. You can then compare those individual moments of inertia to the geometric calculations above.] Click START, and wait for the graphing to start. Release the hanging mass. Try to catch the rotational apparatus so that the "bounce" at the end is not too extreme -- you will be able to get an accurate measurement without worrying about the last few centimeters of the drop. You should get a nice constant positive slope velocity graph (if your rotary sensor is "backwards", your slope might be negative – that's fine, just reset the graph axes so that you can see the slope – and you will want to record any SLOPE measurements as POSITIVE magnitudes).

3. Calculating the acceleration. Select a region of data where the angular velocity curve appears to be a nice straight line. [Note: try to stay away from the "ends" of the linear motion.] Click the FIT button 

, select LINEAR FIT. The fit results box should show the slope, which represents the average angular acceleration. You can also check the accuracy of the fit (the closer the magnitude of the "r" value in the result box is to 1.0, the better the fit). Record those values on the Data/Question sheet. Name this data run (call it "apparatus"). Occasionally save your experiment file (under a different name than the original).

4. Finding the moment of inertia for the apparatus alone. Using the angular acceleration value above, calculate the moment of inertia based on the equation from Section 1.1 and record your results on the Data/Question sheet. (Remember the two "constant" values from the I formula on page 2.)

Prediction Answer on the Data/Question sheet
If you add the flat disk to the rotation apparatus and perform the same procedure, what will happen (increase/decrease/remain same) to the following variables: acceleration, tension, moment of inertia?

5. Measuring the moment of inertia of the disk and apparatus. Place the flat disk on the rotational apparatus and prepare the equipment for graphing as before. You may need to use the Styrofoam padding to keep the disk level on the apparatus - the Styrofoam is so light you can ignore its effect on the measurements. Use 350 grams for the hanging mass and graph the motion as you did before. Select a region of data and use the LINEAR FIT again to find the average angular acceleration, and calculate the moment of inertia (in this case of the disk and the apparatus both). Record this on the Data/Question sheet.

6. Store this run. Store this run and call it "disk".

7. Calculating the moment of inertia of the disk. Subtract the measured moment of inertia of the apparatus from the measured moment of the disk/apparatus to find the moment of inertia for the disk alone. Also find the % difference with the moment of inertia for the disk based on the geometric equations in Section 1.3 and record your calculations on the Data/Question sheet.

Question Answer these on the Data/Question sheet.

a) How do the two values of the moment of inertia compare to each other? What are the largest sources of uncertainty in the variables that lead up to these final values?

Prediction b) Without actually calculating the answer, if you add the hoop to the rotation apparatus instead and perform the same procedure, what will happen (increase/decrease/remain same) to the following variables: acceleration, tension, moment of inertia?

8. Measuring the moment of inertia of the hoop and apparatus. Take the disk off the apparatus, and place the hoop on the rotational apparatus and prepare the equipment for graphing as before. Use 350 grams of hanging mass and graph the motion as you did before. Use the LINEAR FIT to find the average angular acceleration, and calculate the moment of inertia (in this case of the hoop and the apparatus both). Record the calculations on the Data/Question sheet.

9. Store and save. Store data run as "hoop", save this file for future reference.

10. Calculating the moment of inertia of the hoop. Subtract the moment of inertia of the apparatus from the moment of the hoop/apparatus to find the moment of inertia for the hoop alone. Also calculate the moment of inertia for the hoop based on the geometric equations in Section 1.3 and find the % difference:

Question Record these on the Data/Question sheet.

a) How do the two values of the moment of inertia compare to each other? What are the largest sources of uncertainty in the variables that lead up to these final values?

b) Could this method be used to find the moment of inertia of a non-symmetric object? Could it be used to find the moment of inertia of a regular object, but not centered on the axis of the rotation apparatus?

11. Graph the data. Print a graph for the final report. Hide the apparatus run, just show the Hoop run and the Disk run.

DATA/QUESTION SHEET FOR LAB 11 - MOMENT OF INERTIA

Part 1 Introduction to Rotational Inertia

2. Initial measurements. Record the values on the Data/Question sheet.

Make sure you record the **radius** - not the diameter!

$r_{rotation\_apparatus}$ = __________ m

r_{rotary_sensor} = __________ m

Disk	Hoop
R_{disk} = __________ m	R_{inside} = __________ m
	$R_{outside}$ = __________ m
M_{disk} = 4.76 kg	M_{hoop} = 4.08 kg

(The mass values are "rough" measurements)

3. Preparation calculations. Using the values in the boxes above, and the mass of the hanging mass (to start with, you will use 350 grams or 0.35 kg), calculate the two "constants" from the moment of inertia expression on page 2, and record them below:

$$m_{hanging\_mass}\, g \left(r^2_{apparatus} / r_{rotary} \right) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_ \qquad m\left(r_{apparatus}\right)^2 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Part 2 - Dynamic Measurement of Moment of Inertia

3. Calculating the acceleration.

The slope = "m" = angular acceleration (apparatus only) = $\alpha_{apparatus}$ =______ ± ______ rad/sec^2

|"r" value of fit| = ___________ (close to 1 is a very good fit)

4. Finding the moment of inertia for the apparatus alone. Using the acceleration above, calculate the moment of inertia based on the equation from Part 1 and record your results below: (Remember your two values from section 1.3 above.)

$$I_{apparatus} = \frac{\left[m_{hanging\_mass}\, g \left(r^2_{apparatus} / r_{rotary} \right) \right]}{\alpha_{apparatus}} - \left[m_{hanging\_mass} \left(r_{apparatus} \right)^2 \right] = \_\_\_\_\_\_\_\_\_\_ \text{ kgm}^2$$

Prediction If you add the flat disk to the rotation apparatus and perform the same procedure, what will happen (increase/decrease/remain same) to the following variables: acceleration, tension, moment of inertia measured?

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5. Measuring the moment of inertia of the disk and apparatus.

The slope = "m" = angular acceleration (app + disk) = α_{disk_app} =________ ± ________ rad/sec^2

|"r" value of fit| = ___________ (close to 1 is a very good fit)

$$I_{disk\_apparatus} = \frac{\left[m_{hanging\_mass}\, g \left(r^2_{apparatus} \big/ r_{rotary} \right) \right]}{\alpha_{disk\_apparatus}} - \left[m_{hanging\_mass} \left(r_{apparatus} \right)^2 \right] = \_\_\_\_\_\_\_\_\_\_ \text{ kgm}^2$$

7. Calculating the moment of inertia of the disk. Subtract the measured moment of inertia of the apparatus from the measured moment of the disk/apparatus to find the moment of inertia for the disk alone. Record your calculations below:

$$I_{disk(meas)} = I_{disk\_app(meas)} - I_{apparatus(meas)} = \_\_\_\_\_\_\_\_\_\_ \text{ kgm}^2$$

Prediction Without actually calculating the answer, if you add the hoop to the rotation apparatus instead and perform the same procedure, what will happen (increase/decrease/remain same) to the following variables: acceleration, tension, moment of inertia measured?

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____

8. Measuring the moment of inertia of the hoop and apparatus. Use the Analyze tool and find the average angular acceleration for the hanging mass, and calculate the moment of inertia (in this case of the hoop and the apparatus both). Record the calculations below:

The slope = "m" = angular acceleration (app + hoop) = α_{hoop_app} =________ ± ________ rad/sec^2

|"r" value of fit| = ___________ (close to 1 is a very good fit)

$$I_{hoop\_apparatus} = \frac{\left[m_{hanging\_mass}\, g \left(r^2_{apparatus} \big/ r_{rotary} \right) \right]}{\alpha_{hoop\_apparatus}} - \left[m_{hanging\_mass} \left(r_{apparatus} \right)^2 \right] = \_\_\_\_\_\_\_\_\_\_ \text{ kgm}^2$$

10. Calculating the moment of inertia of the hoop. Subtract the moment of inertia of the apparatus from the moment of the hoop/apparatus to find the moment of inertia for the hoop alone.

$$I_{hoop(meas)} = I_{hoop\_app(meas)} - I_{apparatus(meas)} = \_\_\_\_\_\_\_\_\_\_ \text{ kgm}^2$$

11. Calculating % Difference.

$$\text{\% difference for Disk} = \frac{|\, I_{disk(cal)} - I_{disk(meas)} \,|}{1/2(I_{disk(cal)} + I_{disk(meas)})} \times 100 \;; \quad \text{\% difference} = \_\_\_\_\_\_\_\_\_\_$$

$$\text{\% difference for Hoop} = \frac{|\, I_{hoop(cal)} - I_{hoop(meas)} \,|}{1/2(I_{hoop(cal)} + I_{hoop(meas)})} \times 100 \;; \quad \text{\% difference} = \_\_\_\_\_\_\_\_\_\_$$

Box and return equipment to the instructor. Please insure a good experience for the next lab group by cleaning up your lab station.

How do I write up this lab? … What is required for this lab report?

Consult the Rubric for this experiment and the “Lab Report Instructions” document.

Questions/Suggestions → Dr. Changgong Zhou czhou@ltu.edu

Portions of this laboratory manual have been adapted from materials originally developed by Priscilla Laws, David Sokoloff and Ronald Thornton for the Tools for Scientific Thinking, RealTime Physics and Workshop Physics