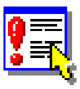
LAB 10 - MOMENT OF INERTIA(TPL1)

Objectives

- To calculate the moment of inertia from the mass and geometric dimensions.
- Measure the moment of inertia of a system with a dynamic method.

Preliminary activities



1. Prepare equipment. Make sure the rotary sensor is connected to the digital adapter, and the adapter is plugged into the Xplorer GLX, then connected the Xplorer GLX to the laptop (please follow the instruction in Blackboard). It would be very helpful to look at the online help page for this lab. Start DataStudio.

2. Load the experiment file. Load the DataStudio file **`Lab10_Inertia.DS`** from the **Tech1** folder.

Part 1 Introduction to Rotational Inertia

Introduction If an unbalanced torque (a net torque) is applied to an object, there will be an overall rotation. The rotation will have an angular acceleration (α) that is proportional to the applied torque (τ), and inversely proportional to the moment of inertia of the object (I) as shown below:

$$\tau = I\alpha$$

We can apply a torque to a rotating turntable by applying tension to string wound around an axis of the turntable. Hanging a mass from the string, over a pulley, creates the tension. The mass is allowed to drop, vertically, and thus accelerates. The linear acceleration of the mass is the same as the linear acceleration of the string, and thus of a point on the axis of the turntable. That linear acceleration of the mass is related to the angular acceleration of the apparatus by a factor of the radius of the axis cylinder. We can measure the angular acceleration of the rotating platform with the rotary sensor. See the diagram and equations below:

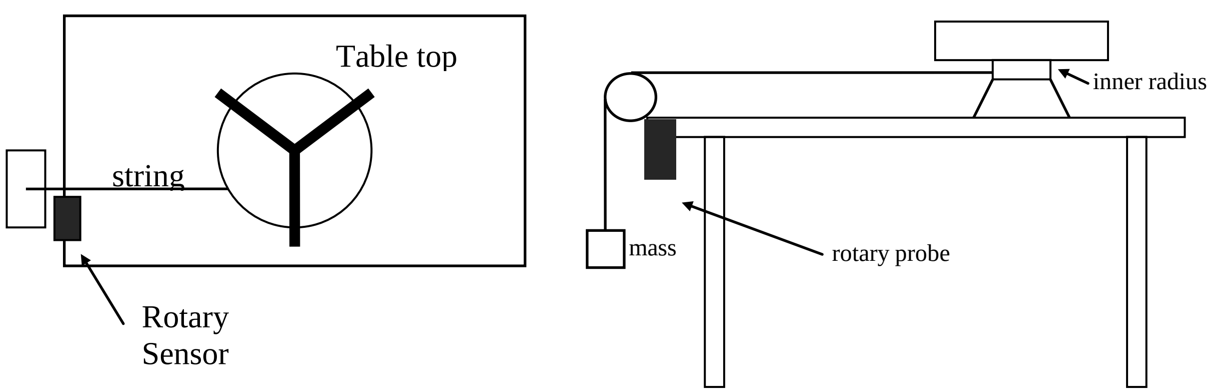


Figure 1 - Top view and side view of rotational inertia apparatus

1. Dynamics calculations. The tension in the string is dependent on the linear acceleration of the mass. That linear acceleration is related to the angular acceleration of the rotary sensor, by the radius of the rotary sensor. We can use DataStudio to measure the angular acceleration of the rotary sensor. Thus, we can generate an expression for the tension in the string dependent on the angular acceleration of the rotary sensor:

Sum of forces on falling mass: downward = mg, upward = T ; downward taken as positive

$$\sum F = ma\ ;\ \ m_{hanging\_mass}a = \sum F\ ;\ \ \sum F = m_{hanging\_mass}g - T$$

$$\sum F = m_{hanging\_mass}a = m_{hanging\_mass}g - T$$

$$T = m_{hanging\_mass}(g - a_{hanging\_mass}) = m_{hanging\_mass}(g - \alpha_{rotary}r_{rotary})$$

because $a_{hanging\_mass} = r_{rotary}\alpha_{rotary}$

The tension in the string is applied to the rotation apparatus at the distance of the inner radius (the radius of the cylinder around which the string is wrapped). This tension creates an unbalanced torque that gives an angular acceleration to the rotation apparatus. The angular acceleration is related to the moment of inertia of whatever is on the apparatus (we will put hoops and disks on the apparatus to create different moments of inertia). That angular acceleration is also related to the linear acceleration of the falling mass by the radius of the inner cylinder. Putting these relationships together (whew!), we can solve for the moment of inertia:

$$\sum \tau = I\alpha \qquad \Rightarrow \qquad I = \frac{\tau_T}{\alpha} = \frac{(T)(r)}{\alpha} = \frac{m_{hanging\_mass}(g - \alpha_{rotary}r_{rotary})r_{apparatus}}{\alpha_{device}}$$

$$\Rightarrow \qquad I = \frac{m_{hanging\_mass}\,g\,r_{apparatus}}{\alpha_{device}} - m_{hanging\_mass}r_{rotary}r_{apparatus}\left(\frac{\alpha_{rotary}}{\alpha_{device}}\right)$$

Note that $\frac{\alpha_{rotary}}{\alpha_{device}} = \frac{r_{apparatus}}{r_{rotary}}$ … thus $\boxed{I = \frac{m_{hanging\_mass}\,g\left(r^2_{apparatus}/r_{rotary}\right)}{\alpha_{rotary}} - m_{hanging\_mass}\left(r_{apparatus}\right)^2}$

[Notice that, for a given hanging mass, the numerator of the first expression and the entire second expression are constant. For each object we rotate (hoop, disk, etc.) we just need to measure the new angular acceleration from the rotary sensor and we can easily calculate the moment of inertia from the above equation. And, the only angular acceleration we measure is from the rotary sensor!]

There are two objects we will put on the rotational apparatus, in various combinations: a flat disk and a hoop. To check the moment of inertia calculation above (from dynamics variables), we can use the geometric definitions of the moments of inertia for these two objects:

$$\boxed{I_{disk} = \frac{1}{2}M_{disk}R^2_{disk} \qquad I_{hoop} = \frac{1}{2}M_{hoop}\left(R^2_{outside} + R^2_{inside}\right)}$$ (geometric formulas)

[NOTE: These "big R" radii are the radii for the disk and the hoop - do not confuse these with the "small r" radii of the rotary sensor or the apparatus itself.]

<u>2. Initial measurements.</u> To be able to use the above equations, we need to measure the inner radius of the rotation apparatus and of the rotary sensor (using a digital caliper), and the dimensions/masses of the hoops and disk. The masses of the hoops/disks are listed on the Data/Question sheets. Use a ruler for the radius measurements on the ring and hoop. Record the values on the Data/Question sheet.

3. Preparation calculations. Using the values from Part 1.2, and the mass of the hanging mass (to start with, we will use 350 grams or 0.35 kg), calculate the two "constants" from the moment of inertia calculation above, and record them on the Data/Question sheets.

Part 2 - Dynamic Measurement of Moment of Inertia

1. Prepare equipment. Make sure that the string is wrapped around the cylinder of the rotation apparatus, and drapes over the rotary sensor (using the OUTER pulley). Using the mass hanger and the slot masses, attach 350 grams (total) to the end of the string, and "wind up" the rotation apparatus, so that the mass starts up near the table edge.

2. Measuring the moment of inertia of the rotation apparatus alone. It would be very difficult to calculate the moment of inertia of the empty apparatus geometrically, so instead we will measure it dynamically. [Later, we will be able to subtract it from the moments we measure with the ring and the hoop to get just the individual moments of inertia of the ring and the hoop. We can then compare those individual moments to the geometric calculations above.] Click START, and wait for the graphing to start. Release the hanging mass. Try to catch the rotational apparatus so that the "bounce" at the end is not too extreme -- we will be able to get an accurate measurement without worrying about the last few centimeters of the drop. You should get a nice constant positive slope velocity graph (if your rotary sensor is "backwards", your slope might be negative – that's fine, just reset the graph axes so that you can see the slope – and we will want to record any SLOPE measurements as POSITIVE magnitudes).

3. Calculating the acceleration. Using the Smart Tool, select a region where the angular velocity curve appears to be a nice straight line. [Note: try to stay away from the "ends" of the linear motion.] Select a LINEAR FIT. The fit results box should show the slope, this represents the average angular acceleration. We can also check the accuracy of the fit (the closer the magnitude of the "r" value is 1.0, the better the fit). Record those values on the Data/Question sheet. Name this data run (called "apparatus"). Occasionally save your experiment file (under a different name than the original).

4. Finding the moment of inertia for the apparatus alone. Using the angular acceleration value above, calculate the moment of inertia based on the equation from Section 1.1 and record your results on the Data/Question sheet. (Remember the two "constant" values from section 1.3 Data/Question Sheet.)

Prediction Answer on the Data/Question sheet
If we add the flat disk to the rotation apparatus and perform the same procedure, what will happen (increase/decrease/remain same) to the following variables: acceleration, tension, moment of inertia measured?

5. Measuring the moment of inertia of the disk and apparatus. Place the flat disk on the rotational apparatus and prepare the equipment for graphing as before. You may need to use the Styrofoam padding to keep the disk level on the apparatus - the Styrofoam is so light we can ignore its effect on the measurements. Use 350 grams of hanging mass and graph the motion as you did before. Use the Smart Tool and the LINEAR FIT to find the average angular acceleration, and calculate the moment of inertia (in this case of the disk and the apparatus both). Record this on the Data/Question sheet.

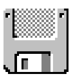
6. Store this run. Store this run and call it "disk".

7. Calculating the moment of inertia of the disk. Subtract the measured moment of inertia of the apparatus from the measured moment of the disk/apparatus to find the moment of inertia for the disk alone. Also find the % difference with the moment of inertia for the disk based on the geometric equations in Section 1.4 and record your calculations on the Data/Question sheet.

Question Answer these on the Data/Question sheet.
a) How do the two values of the moment of inertia compare to each other? What are the largest sources of uncertainty in the variables that lead up to these final values?

Prediction b) Without actually calculating the answer, if we add the hoop to the rotation apparatus instead and perform the same procedure, what will happen (increase/decrease/remain same) to the following variables: acceleration, tension, moment of inertia measured?

8. Measuring the moment of inertia of the hoop and apparatus. Take the disk off the apparatus, and place the hoop on the rotational apparatus and prepare the equipment for graphing as before. Use 350 grams of hanging mass and graph the motion as you did before. Use the linear fit to find the average angular acceleration, and calculate the moment of inertia (in this case of the hoop and the apparatus both). Record the calculations on the Data/Question sheet.



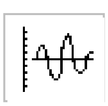
9. Store and save. Store data run as “hoop”, save this file for future reference.

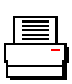
10. Calculating the moment of inertia of the hoop. Subtract the moment of inertia of the apparatus from the moment of the hoop/apparatus to find the moment of inertia for the hoop alone. Also calculate the moment of inertia for the hoop based on the geometric equations in Section 1.4 and find the % difference:

Question Record these on the Data/Question sheet.
a) How do the two values of the moment of inertia compare to each other? What are the largest sources of uncertainty in the variables that lead up to these final values?

b) Could this method be used to find the moment of inertia of a non-symmetric object? Could it be used to find the moment of inertia of a regular object, but not centered on the axis of the rotation apparatus?

11. Graph the data. Print a graph for the final report. Hide the apparatus run, just show the Hoop run and the Disk run.





DATA/QUESTION SHEET FOR LAB 8 - MOMENT OF INERTIA

Part 1 Introduction to Rotational Inertia

2. Initial measurements. Record the values on the Data/Question sheet.

Make sure you record the **radius** - not the diameter!

$r_{rotation\_apparatus}$ = __________ m

r_{rotary_sensor} = __________ m

Disk	Hoop
R_{disk} = __________ m	R_{inside} = __________ m
	$R_{outside}$ = __________ m
M_{disk} = 4.76 kg	M_{hoop} = 4.08 kg

(the mass values are "rough" measurements)

3. Preparation calculations. Using the values in the boxes above, and the mass of the hanging mass (we will use 350 grams or m = 0.35 kg), calculate the two "constants" from the moment of inertia calculation above, and record them below:

$$m_{hanging\_mass}\, g\left(r^2_{apparatus} / r_{rotary}\right) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_ \qquad m\left(r_{apparatus}\right)^2 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

4. Geometric Calculations. Using the values for the Disk and Hoop from above, calculate their moments of inertia.

$I_{disk} = 1/2 M_{disk} R^2$; I_{disk} = _____________kgm^2

$I_{hoop} = 1/2 M_{hoop}(R^2_{outside} + R^2_{inside})$; I_{hoop} = _____________kgm^2

Part 2 - Dynamic Measurement of Moment of Inertia

3. Calculating the acceleration.

slope = "m" = angular acceleration (apparatus only) = $\alpha_{apparatus}$ =________ ± ________ rad/sec^2

|"r" value| = ___________ (close to 1 is a very good fit)

4. Finding the moment of inertia for the apparatus alone. Using the acceleration above, calculate the moment of inertia based on the equation from Section 1.1 and record your results below: (Remember your two values from section 1.4.)

$$I_{apparatus} = \frac{\left[m_{hanging\_mass}\, g \left(r^2_{apparatus} / r_{rotary} \right) \right]}{\alpha_{apparatus}} - \left[m_{hanging\_mass} \left(r_{apparatus} \right)^2 \right] = \_\_\_\_\_\_\_\_\_ \text{ kgm}^2$$

Prediction If we add the flat disk to the rotation apparatus and perform the same procedure, what will happen (increase/decrease/remain same) to the following variables: acceleration, tension, moment of inertia measured? ____

____

____

5. Measuring the moment of inertia of the disk and apparatus.

slope = "m" = angular acceleration (app + disk) = α_{disk_app} =________ ± ________ rad/sec^2

|"r" value| = ___________ (close to 1 is a very good fit)

$$I_{disk\_apparatus} = \frac{\left[m_{hanging\_mass}\, g \left(r^2_{apparatus} / r_{rotary} \right) \right]}{\alpha_{disk\_apparatus}} - \left[m_{hanging\_mass} \left(r_{apparatus} \right)^2 \right] = \_\_\_\_\_\_\_\_\_ \text{ kgm}^2$$

7. Calculating the moment of inertia of the disk. Subtract the measured moment of inertia of the apparatus from the measured moment of the disk/apparatus to find the moment of inertia for the disk alone. Record your calculations below:

$$I_{disk(meas)} = I_{disk\_app(meas)} - I_{apparatus(meas)} = \_\_\_\_\_\_\_\_\_ \text{ kgm}^2$$

Prediction Without actually calculating the answer, if we add the hoop to the rotation apparatus instead and perform the same procedure, what will happen (increase/decrease/remain same) to the following variables: acceleration, tension, moment of inertia measured?

____

____

8. Measuring the moment of inertia of the hoop and apparatus. Use Analyze and find the average angular acceleration for the hanging mass, and calculate the moment of inertia (in this case of the hoop and the apparatus both). Record the calculations below:

slope = "m" = angular acceleration (app + hoop) = α_{hoop_app} =________ ± ________ rad/sec^2

|"r" value| = ___________ (close to 1 is a very good fit)

$$I_{hoop\_apparatus} = \frac{\left[m_{hanging\_mass}\, g \left(r^2_{apparatus} / r_{rotary} \right) \right]}{\alpha_{hoop\_apparatus}} - \left[m_{hanging\_mass} \left(r_{apparatus} \right)^2 \right] = \_\_\_\_\_\_\_\_\_ \text{ kgm}^2$$

10. Calculating the moment of inertia of the hoop. Subtract the moment of inertia of the apparatus from the moment of the hoop/apparatus to find the moment of inertia for the hoop alone.

$$I_{hoop(meas)} = I_{hoop\_app(meas)} - I_{apparatus(meas)} = \_\_\_\_\_\_\_\_\_\_ \text{ kgm}^2$$

11. Calculating % Difference.

$$\text{\% difference for Disk} = \frac{| I_{disk(cal)} - I_{disk(meas)} |}{1/2(I_{disk(cal)} + I_{disk(meas)})} \text{ x } 100 \text{ ; } \quad \text{\% difference} = \_\_\_\_\_\_\_\_\_\_$$

$$\text{\% difference for Hoop} = \frac{| I_{hoop(cal)} - I_{hoop(meas)} |}{1/2(I_{hoop(cal)} + I_{hoop(meas)})} \text{ x } 100 \text{ ; } \quad \text{\% difference} = \_\_\_\_\_\_\_\_\_\_$$

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