LAB 7 BEAM STRUCTURE - BENDING AND BREAKING

Goals:

- Measure the force necessary to bend beams of wood and metal to a given deflection
- Measure forces necessary to bend and then break different lengths/diameters of spaghetti

Part 1 Simple Bending Theory

In this lab, we will be applying a simple load (a force) to a beam made out of wood, steel, or spaghetti. The force will be applied in the center of the beam, with the beam "held" by two hooks near the end. This is an example of a simply loaded beam, or a beam with a concentrated load. In this case, the segment of the beam where the force is applied is small compared to the whole length of the beam.

Looking at the basic bending theory, we can construct a picture of the bending of a beam, and the equations that should govern it. Suppose we look at a beam that is bent in a curve perpendicular to its length:

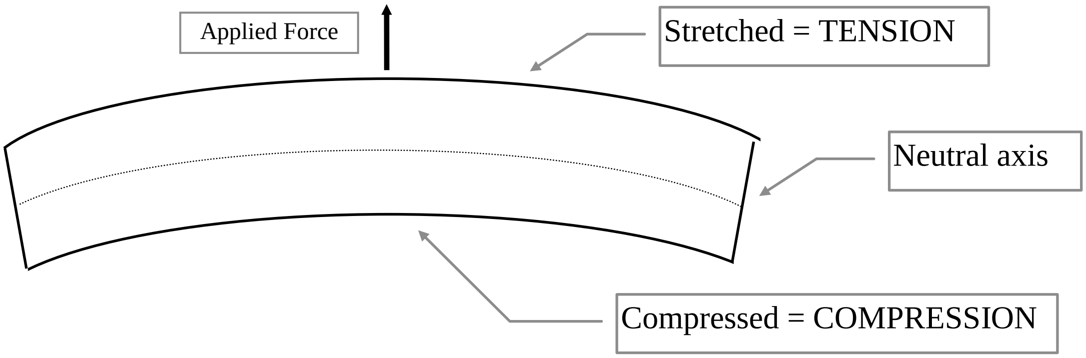


Figure 1 - Compression and Tension in a bent beam

If the beam is relatively uniform (in composition) and with the applied force as shown above, the beam can bend in a radial curve. The top part of the beam (in the picture) will be stretched, and the bottom part of the beam will be compressed. Thus, the outside of the curve is under tension (force applied toward the ends of the beam and parallel to the axis) and the bottom is under compression (with the force applied parallel to axis, but inward from the ends). There is also a "neutral axis" that is under neither tension nor compression. The following theory is based on this picture.

Part 2 Calculation of Force vs. Displacement relationship

There is a stress (the ratio of the force applied to the cross-sectional area) and a strain (the ratio of the change in length to the original length). Simple bending theory also assumes that a material has a Young's Modulus (ratio of stress over strain) that is the same in tension or in compression situations. The Young's modulus values are given below for the beams that we will use - the modulus depends on the type of material the beam is made of. There is another variable used in different bending situations called the Second Moment of the beam. These calculations will be given below, and depend on the dimensions of the beam.

Applying the standard beam bending theories, we can find a relationship between the force applied to the beam and the deflection of the beam. In this case, we will use F for the force applied, and d for the displacement. In this simple case, the relationship is as follows:

$$\frac{F}{d} = \frac{48EI}{L^3}$$

There are some important features of this equation as it relates to our current investigation:

- The force is proportional to the distance (as in Hooke's spring law)
  (we should expect a straight line curve if we plot F vs. d)
- The Young's modulus (E) depends on the material
- The second moment (I) depends on the cross-sectional dimensions
- The force vs. distance ratio is inversely proportional to the length (L) cubed

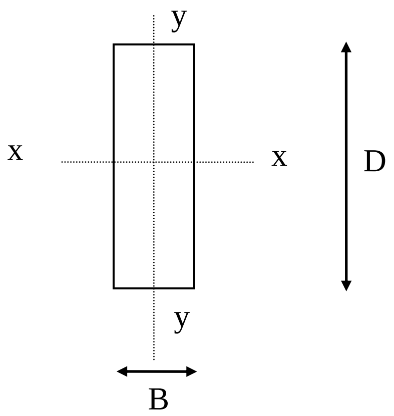
We are using circular rods of wood and metal, and then circular pieces of spaghetti. Finally, we will use a flatter noodle (the linguini), which we can approximate with a rectangular formula. The second moments are given below:

$$I_{circular} = \frac{\pi D^4}{64} \qquad \text{D=diameter}$$

$$I_{rectangular} : \; I_{\perp thin} = \frac{BD^3}{12} \quad I_{\perp wide} = \frac{B^3 D}{12}$$

depending if bent $\perp$ to thin side or wide side

(We will use the rectangular moments to explain what happens with the linguini!)



The Young's Moduli for the wood and the steel are given below (the spaghetti is obviously not listed in the books!):

$$E_{wood} \approx 2x10^{10}\ Pa \ \text{(empirical)} \quad E_{metal} \approx 20x10^{10}\ Pa \quad \text{so,} \ E_{metal}\ \alpha\ 10E_{wood}$$

So, with the materials we will be using, what relationships should be investigated?

LENGTH: We will have two different lengths of the wood dowels (equal second moments and Young's moduli). We can test the length-cubed relationship. We should find the ratio of F/d to be small for the long (3 ft) piece of wood, and large for the short (1.5 ft) piece of wood. These ratios should be in the range from 100-4000 N/m. The F/d ratio for the 1.5-foot should be roughly 8 times larger than the F/d ratio for the 3-foot (2^3= 8) - so we will compare the cube root (of the ratio of the two F/d values) to the value of 2.

$$\sqrt[3]{\left.\frac{F}{d}\right|_{1.5\ foot} \Big/ \left.\frac{F}{d}\right|_{3\ foot}} \approx 2 \ ?$$

WOOD vs. METAL: The metal rods are thinner than the wooden rods. Thus there is a ratio in the diameters. That means a we will have a "fourth power of the diameter ratio" for the second moments. The term EI shows up in the F/d ratio and the E (metal) is 10 times the E (wood). Multiplying these, we find that the EI variable for the metal rod will be related to 10 times the diameter to the fourth power compared to the wood (since the two rods we will compare are both 3 feet in length). Thus the "fourth root" (or square root and square root again) of the F/d ratios should give 1.778 times the diameters.

$$\sqrt[4]{\left.\frac{F}{d}\right|_{metal} \Big/ \left.\frac{F}{d}\right|_{wood}} \approx (10)^{1/4}\left(\frac{D_{metal}}{D_{wood}}\right) = 1.778\left(\frac{D_{metal}}{D_{wood}}\right)?$$

SPAGHETTI The thin vs. thick relationship can be approximated using the above equations.

LINGUINI We should find the linguini to be much easier to bend perpendicular to its wide side as opposed to perpendicular to its thin side (depending on the time allowed, you could try to apply the measurements of its width and thickness to the second moment equations above to see what the ratio in I values would be).

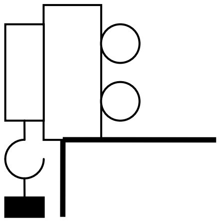
Part 3 Initial preparation

Preliminary activities

1. Prepare equipment. Connect the motion detector and the force probe to the Xplorer GLX, and then to your laptop. It would be very helpful to look at the online help page for this lab. The force probe should be attached to the cart. Start the DataStudio software.

2. Load the experiment file. Load the file `Lab07_BeamBending_Inverted.ds` from **Physics1** folder.

3. Prepare the force probe. Press the ZERO button on the side of the probe (this is a "mechanical" tare for the probe). We can't "zero" the probe in DataStudio, so occasionally we might need to keep track of a zero "offset" and account for it.



4. Beam bending setup. See the online help page for pictures illustrating how to set up the equipment. The support hooks for the beams should be at a vertical height such that the string from the force probe will be horizontal. The motion detector should be set up so that it can detect about 20 cm of the motion of the cart (but keep it close enough that we don't have to worry as much about the alignment of the detector). Remember the motion detector must be at least 50 cm from the object it is measuring.

Part 4 Wooden Dowels - Bending

1. Diameters. There should be two wooden dowels and one metal dowel on the lab table. Using the micrometer, measure the diameters of the rods in several places to determine the average diameter (take the readings in metric units - millimeters). Record these measurements on the Data/Question sheet.

2. Positioning the wooden dowel. Start with the 3-foot wooden dowel. The string connected to the force probe has a loop on the other end. Slip the loop over the wooden dowel and center the dowel between the hooks that are the farthest apart. Center the string loop on the dowel. There may be a mark on the dowel to help the centering process. Finally, center the horizontal rod vertically, so that the string coming off the force probe, when taut, will be horizontal. Make sure that the track is centered and the cart can move smoothly back.

3. Testing the motion detector. Disconnect the string from the force probe and start the graphing program to make sure the motion detector can pick up the cart's motion. Set the upper scale reading on the distance just greater than the farthest distance that the motion detector can pick up. (Make sure the motion detector is 'strapped' to the track with a rubber band.)

Getting a good Force/Distance graph:

Here is a trick to help ensure a good Tension/Distance graph – keep the string connected to the force probe under tension **at all times** when you are taking data. This means you start collecting data with the string under tension, and it is still under tension when you stop taking data. (See, if the string is loose, the distance can change, but the force still is zero ... these data points obviously don't fit our proportional relationship, and will mess up the "curve fitting" we are going to use.)

4. Measuring the flex of the wooden dowel. Reconnect the string to the end of the force probe and move the force cart back so that the string is taut. Click START. Slowly roll the force probe back from the dowel, to create more tension in the string, and an applied force on the center of the dowel. Apply an increasing force until the dowel bends a few centimeters, and then slowly release the tension back close to **not quite** zero, and continue in cycles like that [keep **some** tension at all times!]. Watch the Force vs. Distance graph; you should see a straight-line relationship (relatively straight). An example of the force and distance vs. time is shown on the online help page. Don't let the string lose tension until you are finished collecting. Repeat until you find a nice straight-line graph for the Force vs. Distance.

Note: DO NOT TRY TO BREAK THE WOODEN DOWEL!!

2. Making a linear fit. Click on the FIT tool button (next to the "calculator" button). Check the LINEAR option. Now, in the graph, you should find a textbox that shows the numbers for the Fit. See the example on the next page:

5. Finding a linear fit (for 3 foot wood). Click once in the Force vs. Distance graph (to make it active). Click on the FIT tool button (next to the "calculator" button). This should create an annotation indicating the slope and intercept, and statistical correlation (the "r" value). We don't care about the intercept, but the slope is the proportional factor we need to record. (The closer the correlation is to magnitude 1, the better the fit.) Record the numbers obtained in the annotation on the Data/Question sheet.

Questions Answer on the Data/Question sheet.
a) Is the straight line fit a good fit? (Look at the "r" value in the fit results ... the closer it is to 1.0 the better the fit {you should have at least 0.95 for a decent fit})?
b) How straight is the complete force vs. distance graph? What reasons could you think of for the line not to be perfectly straight?
c) We only have one rod's information so far, but we still have an indication that the bending theory should be reasonable, what is that evidence?

Prediction Suppose you had a length of the wooden dowel that was shorter than the one you are using now. How would you expect the shapes of the graphs to change (force relative to this dowel, slope relative to this dowel)?

6. Rename your data. Rename this data run for future comparison (call it "wood 3ft"). You could also hide the data run so it won't confuse us. (Save the experiment for safekeeping.)

7. Linear fit of the 1.5-foot wooden dowel. Repeat the above procedures with the smallest length wooden dowel (1.5 foot). Reset the eyehooks for this new length. Record a similar Force vs. Distance graph and calculate the slope based on a fit of the graph. Record the calculations and comparisons on the Data/Question sheet. Store this data run also (name it "wood 1.5ft") and hide it. (Save the experiment for safekeeping.)

Part 5 Metal Rods - Bending

Prediction Answer on the Data/Question sheet
The metal rod has a smaller diameter than the wooden dowels. Flex the rod (and the two dowels) slightly and make a prediction about the relative size of the forces on the metal rod, and

the slope of the Force vs. Distance curve. Discuss this with the members of the group and come up with a "group prediction".

1. Metal rod. Repeat the procedure from Part 3 with the metal rod in place of the wooden dowel. Do not exert too much force on the metal rods - we don't want to overstress them and bend them out of shape for future lab sections. Find the linear fit for the force vs. distance graph. Record your information on the Data/Question sheet. Store this run as "metal 3ft", and save the experiment file.

Questions Answer on the Data/Question sheet

a) How does the slope of the metal rod graph compare to the slopes of the wooden dowel graphs? How well does this match your prediction? Explain.

b) Consider the above information - is the bending theory reasonably supported?

2. Metal vs. Wood comparison. Perform the calculations and comparisons on the Data/Question sheet.

3. Print one graph for the report. Consider the "wood - 3ft vs 1ft" graph from Part 4 – and the "wood vs metal – 3ft" graph from Part 5 – pick the "best" of these two graphs and print it for the lab report.

Part 6 Spaghetti - Bending

1. Different diameters of spaghetti. There are three types of noodles available: thin spaghetti, thick spaghetti, and linguini. Examine the three types of noodles. Gently flex the noodles (but try not to break them). BE CAREFUL WHEN FLEXING THE SPAGHETTI, IF IT BREAKS, FRAGMENTS COULD FLY AROUND - DON'T FLEX IT TOWARD SOMEONE!

Question Answer on the Data/Question sheet.

a) What are your general impressions about the relative force needed to stress all three of the different spaghetti noodles?

b) The linguini is a flatter noodle than the spaghetti. Do you notice anything different about the force to stress the linguini compared to the force needed to stress the regular spaghetti? Did you try to bend it in the plane of the flat side of the noodle? How does that compare to bending it perpendicular to the flat side of the noodle?

c) How would you rank them in order of increasing force?

2. New setup. Reset the eyehooks for this new length. Make sure that the string to the force cart will be horizontal when attached to the spaghetti.

3. Measuring force on Thin Spaghetti. Put a piece of thin spaghetti into the experimental setup. Slowly increase the force on the spaghetti until the spaghetti breaks. Repeat this with 3 or 4 separate strands (you should be able to reload the setup while the time is still running ... that way we can compare the breaking forces of several strands at the same time).

4. Examine the maximum forces. Use the Smart Tool to record some of the values of the breaking force necessary. Record these answers on the Data/Question sheet.

Question Answer on the Data/Question sheet.

Are the values for the breaking force reasonably constant? (If so, find the average of the values and set the vertical axis to just above that value.) What is the average?

5. Bending the thin spaghetti. Reload the experimental setup with another piece of thin spaghetti. We will try to investigate the same force vs. distance relationship as the wooden and metal rods. This will obviously require a much more gentle force than the wooden dowel and metal rod. Start collecting data and gently flex the spaghetti. Look for the straight-line relationship on the Force vs. Distance graph. [This will obviously not be as "clean" as the metal and wooden dowels!]. Before the time runs out, pull slowly but steadily, enough to break the strand (so that we can make sure this fits in with the average obtained before - to ensure this is a 'reasonable' strand). Name this run (“thin spaghetti”) and record your answers on the Data/Question sheet.

Question Answer on the Data Question sheet.
Can you see a very rough "straight line" trend to the data on the Force vs. Distance graph? How does the slope of this force vs. time curve compare to the wooden rods and metal rods? Is this to be expected {consider your hand bending of the spaghetti and the metal or wooden rods}?

Prediction What would you expect the results would be like for the thick spaghetti and the linguini (in both orientations)?

6. Thick spaghetti. Repeat the above steps for the thick spaghetti. Name this run (“thick spaghetti”) and record your answers on the Data/Question sheet.

Question Answer on the Data Question sheet.
How does the slope of this force vs. time curve compare to that of the thin spaghetti? Does this match your prediction earlier?

7. Linguini. Repeat the above steps for the linguini. Make sure you try both orientations of the linguini. Name the runs, rename them, and record your answers on the Data/Question sheet. Save the experiment file.

Question Answer on the Data Question sheet.
How does the slope of this force vs. time curve compare to that of the two spaghetti curves? Does this match your prediction earlier?

DATA/QUESTION SHEET FOR LAB 7 BEAM STRUCTURE - BENDING AND BREAKING

Part 4 Wooden Dowels - Bending

1. Diameters.

Diameters

Type of rod	Trial 1	Trial 2	Trial 3	Trial 4	Average
metal (3’)					
wood (3’)					
wood (1.5’)					

5. Finding a linear fit (for wood - 3ft).

Slope of (wood – 3ft) F vs. d graph = _________ ± _________ N/m

Questions a) Is the straight line fit a good fit? (Look at the “r” value in the fit results ... the closer it is to 1.0 the better the fit {you should have at least 0.95 for a decent fit})?

____

____

b) How straight is the complete force vs. distance graph? What reasons could you think of for the line not to be perfectly straight?

____

____

c) We only have one rod's information so far, but we still have an indication that the bending theory should be reasonable, what is that evidence?

____

Prediction Suppose you had a length of the wooden dowel that was shorter than the one you are using now. How would you expect the shapes of the graphs to change (force relative to this dowel, slope relative to this dowel)?

____

____

7. Linear fit of the 1.5-foot wooden dowel.

Slope of (1.5 foot wood) F vs. d graph = _________ ± _________ N/m

Ratio $\left.\frac{F}{d}\right|_{1.5\,foot} \Big/ \left.\frac{F}{d}\right|_{3\,foot}$ = ________ $\sqrt[3]{\text{"that ratio"}}$ = ______ % error _________

Questions a) Find the ratio of the wood-3ft slope and the wood-1.5ft wood slope and take the cube root - how well does that compare to the number 2? Find the percent error - does this reasonably confirm the bending theory?

____

____

Part 5 Metal Rods - Bending

Prediction The metal rod has a smaller diameter than the wooden dowels. Flex the rod (and the two dowels) slightly and make a prediction about the relative size of the forces on the metal rod, and the slope of the Force vs. Distance curve. Discuss this with the members of the group and come up with a "group prediction".

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____

1. Metal rod.

Slope of entire F vs. d graph = _________ ± _________ N/m

Is this a good fit? ("r" value?) __________

Questions a) How does the slope of the metal rod graph compare to the slopes of the wooden dowel graphs? How well does this match your prediction? Explain.

____

____

2. Metal vs. Wood comparison.

Find the ratio $\left.\frac{F}{d}\right|_{metal} \bigg/ \left.\frac{F}{d}\right|_{wood}$ = _________ (a) "Fourth root" of (a) = __________ (b)

"other side" ratio = $(10)^{1/4}\left(\frac{D_{metal}}{D_{wood}}\right)$ = _________ (c) % difference between (b) and (c) = __________

b) Considering the above information - is the bending theory reasonably supported?

____

____

Part 6 Spaghetti - Bending

1. Different diameters of spaghetti.

Question a) What are your general impressions about the force needed to stress all three of the different spaghetti noodles?

____

____

b) The linguini is a flatter noodle than the spaghetti. Do you notice anything different about the force to stress the linguini compared to the force needed to stress the regular spaghetti? Did you try to bend it in the plane of the flat side of the noodle? How does that compare to bending it perpendicular to the flat side of the noodle?

____

____

c) How would you rank them in order of increasing force?

4. Examine the maximum forces. Use the Smart Tool to record some of the values of the breaking force necessary.

Breaking forces ________ ________ ________ ________ ________

Average breaking force = ________ N

Question Are the values for the breaking force reasonably constant? (If so, find the average of the values and set the vertical axis to just above that value.) What is the average?

5. Bending the thin spaghetti.

Slope of good section of the F vs. d graph = ________ ± ________ N/m

Question Can you see a very rough "straight line" trend to the data on the Force vs. Distance graph? How does the slope of this force vs. time curve compare to the wooden rods and metal rods? Is this to be expected {consider your hand-bending of the spaghetti and the metal or wooden rods}?

Prediction What would you expect the results would be like for the thick spaghetti and the linguini (in both orientations)?

6. Thick spaghetti.

Breaking forces ________ ________ ________ ________ ________

Average breaking force = ________ N

Slope of good section of the F vs. d graph = ________ ± ________ N/m

Question How does the slope of this force vs. time curve compare to that of the thin spaghetti? Does this match your prediction earlier?

7. Linguini.

Bend “easy way” - Slope of good section of the F vs. d graph = _________ ± _________ N/m

Bend “hard way” - Slope of good section of the F vs. d graph = _________ ± _________ N/m

Question How does the slopes of these force vs. time curves compare to that of the two spaghetti curves? Does this match your prediction earlier?

____

____

How do I write up this lab? … What is required for this lab report?

Consult the Rubric for this experiment and the “Lab Report Instructions” document (both found on the Lab Schedule page).

Questions/Suggestions -> Dr. Scott Schneider - S_SCHNEIDER@LTU.EDU

Portions of this laboratory manual have been adapted from materials originally developed by Priscilla Laws, David Sokoloff and Ronald Thornton for the Tools for Scientific Thinking, RealTime Physics and Workshop Physics curricula. You are free to use (and modify) this laboratory manual only for non-commercial educational uses.